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The Impact of Regret on the Demand for Insurance

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Abstract

We examine optimal insurance purchase decisions of individuals that exhibit behavior consistent with Regret Theory. Our model incorporates a utility function that assigns a disutility to outcomes that are *ex post* suboptimal, and predicts that individuals with regret-theoretical preferences adjust away from the extremes of full insurance and no insurance coverage. This prediction holds for both coinsurance and deductible contracts, and can explain the frequently observed preferences for low deductibles in markets for personal insurance.

Introduction

In models of insurance demand that are based on expected utility, it is well established that a risk-averse individual will purchase full insurance when the insurance contract is fairly priced and less than full insurance with a positive proportional loading factor (Mossin, 1968; Schlesinger, 1981, 2000). This result applies to insurance policies of both a coinsurance structure (where indemnity is a proportion of the loss) and a deductible structure (where indemnity is a payoff for losses above a certain threshold).[1] The supply of insurance in these models is assumed to be exogenous and individuals' preferences are assumed to be consistent with Expected Utility Theory (EUT).

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[1] Practitioners in the insurance industry often define coinsurance as a minimum amount of the total value of a property that must be insured. Our use of the term "coinsurance" is, however, consistent with its usage in the academic literature.

Given the plethora of observed deviations from EUT (e.g., Allais Paradox, preference reversals), it is reasonable to expect that observed insurance decisions would not strictly adhere to the predictions of EUT-based models. The degrees of robustness of these predictions when the assumption of an EU-maximizing insurance customer is relaxed are mixed. For example, Machina (2000) shows that some predictions of EUT-based insurance demand models remain valid as long as the individual is risk averse; conditions of outcome convexity and linearity in probabilities are unnecessary for both coinsurance and deductible contracts. Doherty and Eeckhoudt (1995) found when applying the "Dual Theory" of Yaari (1987) (where expected utility is not linear in probabilities), that only corner solutions (full insurance or no insurance) can be optimal for coinsurance contracts and that optimal solutions differ from the EUT model (and may be interior) for deductible contracts. Approaches based on the Savage (1954) "minimax regret" rule have also attempted to explain insurance demand.[2] Razin (1976) found that minimax regret predicted preferences for positive deductibles even when insurance is fairly priced. Briys and Louberge (1985) showed that decision makers using the Hurwicz criterion (where decision makers are assumed to assign subjective weights to the best and worst possible outcomes) may prefer either zero or positive deductibles at any loading factor. Clearly, the assumptions on the structure of the utility function and decision rules matter when determining the optimal insurance level.

In this article, we propose an alternative, axiomatically supported utility function that explains how a "regret-averse" individual would hedge risk differently from an EU-maximizing individual. This function, derived from Regret Theory as proposed by Loomes and Sugden (1982) and Bell (1982) and axiomatized by Sugden (1993) and Quiggin (1994), defines regret as the disutility of not having chosen the *ex post* optimal alternative. Behavior of this sort has been observed at various levels of significance in both the laboratory and the field, and we believe that the presence of this behavior may help explain anomalies in the demand for insurance. The major difference between our model and the decision rules discussed in the previous paragraph is that Regret Theory implies that a second attribute (regret) is considered along with wealth as part of the objective function. The previous attempts at this subject assume only one attribute: wealth in the case of EUT, non-EUT, and Dual Theory, and gaps between actual and *ex post* optimal outcomes in the case of minimax regret. The present approach assumes that both wealth and regret play a part in the decision-making process, as does the

[2] One of the earliest models of decision making is the "minimax regret" decision rule of Savage (1954), which incorporates preferences to minimize exposure to *ex post* suboptimal outcomes. This model defines regret as the difference between the outcome of each decision alternative and the best possible outcome among all alternatives for each ultimate state of the world. A decision maker applying minimax regret would determine the highest level of regret that could possibly occur for each decision alternative (among all possible outcomes), and then choose the decision alternative with the lowest of these maximum regret levels (Zeelenberg, 1999). The minimax regret rule is useful because it places an upper bound on the regret that one would experience. However, it does not take into account the probabilities of any of the states of the world occurring. A decision maker could potentially pass up an alternative that would offer a near-certain chance of low regret because of a possibility of an occurrence a low probability event that would induce a large amount of regret.

probability distribution on possible states of the world. In our model an individual maximizes the expected value of the utility function described above. We call this objective function "regret-theoretical expected utility" (RTEU).

Our results predict that preferences that are consistent with the Regret Theory axioms of Sugden (1993) and Quiggin (1994) (i.e., preferences that are "regret-theoretical") should lead to insurance decisions that are "less extreme" than those that would be derived from preferences consistent with the EUT axioms of Savage (1954). This means that when large amounts of insurance are predicted by EUT, a regret-averse individual (i.e., an individual with regret-theoretical preferences) should choose less insurance, and when small amounts of insurance are predicted to be optimal by EUT, the regret-averse individual prefers to purchase more insurance. These results hold for insurance contracts of both the coinsurance form and deductible form, and the direction of the impact of regret depends on the proportional loading factor on the premium of these contracts. We also find that the "critical loading factor" at which the impact of regret goes from positive to negative is independent of the level of regret incorporated in the model. In essence, we show that individuals with regret-theoretical preferences would tend to "hedge their bets," taking into account the possibility that their decisions may turn out to be *ex post* suboptimal. For loading factors that are sufficiently high, these predictions can help explain the preferences for low deductibles that are well known in the study of insurance.

In the next section, we present the RTEU function and describe its properties in light of previous axiomatic and experimental research. In the "The Insurance Model" section, we show how the maximization of RTEU yields optimal insurance choices that are less extreme than those predicted by the conventional expected utility model. In the "Discussion" section, we discuss implications that the overall results have for both the insurance industry and regulatory bodies, including an explanation for observed preferences for low deductibles. All proofs are given in the Appendix.

REGRET-THEORETICAL EXPECTED UTILITY

The key assumption of our model is that individuals avoid the unfavorable consequences of experiencing an outcome that is worse than the best that could have been achieved *had the amount of the loss (or lack thereof) been known in advance*. For example, if an individual purchases little insurance, and then incurs a large loss, the individual would experience some additional disutility of not having purchased more insurance *ex ante*. One can imagine such an individual "kicking oneself" for not having bought more insurance. Had the individual taken more insurance (in the form of a higher coinsurance rate or lower deductible level), not only would the insurance contract have covered more of the loss, but this individual would have "felt better" by having made the "right decision." Among the drivers of this behavior might be the avoidance of emotional regret ("I'll feel regret if I don't have enough insurance") or the need for legitimation ("If I have an accident, I'll have to explain why I didn't buy more insurance"). Regardless of whether this disutility is derived from avoiding either a negative emotion or additional negative consequences, we call this disutility "regret" and can reasonably believe a story of "regret avoidance" in the insurance context.

The RTEU Function

The theoretical root of our approach is in Regret Theory, as formulated by both Bell (1982, 1983) and Loomes and Sugden (1982). Each suggested that decision makers optimize the expected value of a "modified" utility function of the form

$$u(x, y) = v(x) + g(v(x) - v(y)). \tag{1}$$

This model applies to the case of two alternatives: x is chosen by the decision maker and y is not (y is the "foregone" alternative). $v(\cdot)$ is a *traditional* Bernoulli utility function over monetary positions with $v' > 0$ and $v'' < 0$. $g : \mathbb{R}_+ \rightarrow \mathbb{R}$ is a regret function that depends on the difference between the realized value of the chosen alternative and the realized value of the foregone alternative. Note that $g(\cdot)$ represents the regret or rejoicing that the decision maker experiences as a result of receiving x versus not receiving y. If it turns out that $x > y$, then the decision maker made the "correct" choice and gains some additional utility by having passed up the foregone alternative. If $x < y$, then the decision maker experiences disutility from having forgone the possibility of doing better by having chosen the foregone alternative. Thus, Regret Theory assumes not only that decision makers experience regret, but also that the anticipation of experiencing regret is factored into the decision-making processes (Larrick and Boles, 1995).

In the context of insurance, however, an individual chooses from more than two alternatives. Loomes and Sugden (1982) handle the multiple action case by comparing the payoffs from the chosen action against the payoffs from each alternative action separately. In contrast, we compare the payoff from the chosen action against the highest payoff that one could accrue in each possible state of the world. In other words, for different states, we evaluate the chosen action against potentially different alternatives, depending on which alternative yields the highest payoff for that particular state.

Therefore, we introduce a modification of the formulation in (1) by proposing a two-attribute utility function of the form

$$u(w) = v(w) - k \cdot g(v(w^{\max}) - v(w)), \tag{2}$$

where v and g are defined as above, with $g' > 0$, $g'' > 0$, and $g(0) = 0$. We replace x and y with only two of the multitude of possible states: w is the wealth level that is actually realized and $w^{\max}$ is the wealth that the individual could have received by having made the optimal choice with respect to the realized state of nature (the amount of the loss). Since this optimal choice could be different for different states, $w^{\max}$ is a random variable. We also introduce k as the "regret coefficient," a linear weight placed on the regret component of this utility function.[3] If $k = 0$, the individual is a *traditional* risk-averse expected utility maximizer. If $k > 0$, then the utility function of the individual

[3] This utility function is essentially a linear combination of a value for wealth and a value for regret. One could rescale the coefficients on $v(w)$ from 1 to β and on $g(\cdot)$ from k to $(1 - \beta)$, where $\beta = \frac{1}{k+1}$. This transformation places linear weights on two attributes (monetary utility and regret), and allows the decision maker to maximize the resulting function. In this article, we have chosen to use k instead of β purely for mathematical convenience.

includes some compensation for regret and we call the individual *regret averse*. Any preferences that can be represented by (2) are said to be *regret theoretical*. Under these assumptions, all individuals that we consider in this analysis are risk averse (because of the concavity of v), but only those for whom $k > 0$ are also regret averse.

Applying an expected utility representation to (2), we introduce the concept of RTEU, represented by

$$\text{RTEU} = E[v(w) - k \cdot g(v(w^{\max}) - v(w))], \tag{3}$$

where the expectation is taken over the subjective probability distribution of future states of the world. The notion of expected utility that we use here is more akin to that of Savage (1954) than of von Neumann–Morgenstern, in that we are comparing preferences for actions, assuming subjective probabilities, rather than bets on lotteries given known probabilities. The only difference is that the utility for which we are taking the expected value is represented by (2), as opposed to (1). It is clear, however, that an individual who maximizes expected utility as represented by Equation (3) considers the *ex post* payoff of foregone actions on the *ex post* utility of the chosen one. Thus, the Savage axioms for subjective expected utility cannot be represented by an expected utility function of this form.

Nevertheless, (3) is consistent with the Regret Theory axioms of Sugden (1993) and the Axiom of Irrelevance of Statewise Dominated Alternatives (ISDA) proposed by Quiggin (1994). ISDA requires the decision maker to ignore any actions in the feasible set that are statewise dominated by other actions in the set. The critical consequence of Quiggin's ISDA is that if a decision maker's preferences are consistent with ISDA and the Sugden axioms, then the regret associated with a given action in a particular state of nature depends only on the actual outcome and the best possible outcome that the individual could have attained in that same state of nature. And since the Sugden axioms are essentially a reformulation of those of Savage (1954), we have a normative basis for RTEU that allows us to use the model to analyze insurance choices. This result also allows us to focus solely on "regret" and its associated disutility, as opposed to earlier formulations of Regret Theory that also allow for "rejoicing" when the "better" outcome is chosen for the eventual state of the world. In fact, because $g(0) = 0$ and because one can never do better than the best possible outcome, we have eliminated "rejoicing" from the regret/rejoice model altogether. We can then restrict the regret coefficient $k \geq 0$ as measure of the influence of regret on the decision.

Behavioral Characteristics

We have defined regret as the disutility an individual experiences from the value gap between an actual outcome and the best possible outcome that one could have attained in a particular state of nature. Naturally, the selection of an RTEU-maximizing decision involves some trade-off between maximizing the value from the actual outcome and minimizing regret. Any evidence that individuals behave in a regret-avoiding way is consistent with the RTEU function, and for our purposes, regret-theoretic behavior is any behavior that can be modeled by an expected utility function of the RTEU form. In the psychology literature, however, regret often has specific meaning as an emotion or driver of behavior that we do not consider explicitly in this article.

There are some behavioral constructs that are often considered as part of Regret Theory, but are *not*, in our context, consistent with RTEU. First, we emphasize that "regret" is not the same as "disappointment." Zeelenberg et al. (2000) make the distinction clear. "Regret is assumed to originate from comparisons between the factual decision outcome and a counterfactual outcome that might have been *had one chosen differently*; disappointment is assumed to originate from a comparison between the factual decision outcome and a counterfactual outcome that might have been *had another state of the world occurred*" (p. 529). The distinction between disappointment and regret can be examined in terms of the reference point against which the decisions are made Loomes (1988). When a decision maker minimizes regret, the reference point for each action is the outcome that is accrued in the same state of the world from the *other* action. If a decision maker minimizes disappointment, each action is measured against the *ex ante* expectation of what the payoff could have been for the same action. In the insurance context, we are assuming that the disutility is derived from an individual's emotions after making the wrong decision. The *ex post* assessment we are considering is "I should have bought more (or less) insurance" and not "I wish I hadn't incurred that loss." Thus, our interest is clearly in regret and not disappointment.

Second, we note that there is a difference between avoiding regret by making regret-avoiding decisions, and avoiding regret by suppressing regret-inducing information about the outcome of the foregone alternative. Regret-avoiding decisions are of the type we consider in the present work—a subject maximizes the RTEU function. Suppressing regret-inducing information implies that if all signals regarding the eventual state of the world were withheld—and the outcome that would have been experienced had the foregone action been chosen—then the individual could not possibly experience any disutility from that action. Bell (1983), in fact, defines a regret premium as the amount of money a decision maker is willing to pay to cancel, or suppress the knowledge of the outcome of, a foregone lottery. In our context, we are concerned only with the first approach to avoiding regret: making appropriate *ex ante* decisions. It would not be realistic to model a world in which one does not know the amount of the loss that one incurs.

Finally, we emphasize that regret avoidance does not necessarily imply either risk-seeking or risk-avoiding behavior. One could be regret averse and either risk averse or risk seeking. The separation of these two concepts is explained by Zeelenberg (1999), who summarizes an experiment (and two subsequent variations) that offer confirmatory evidence that individuals attempt to avoid regret. In the first, subjects are asked to choose between two gambles, one being "risky" and the other being "safe." In the first condition of the experiment, the outcome of the risky gamble is revealed, and in the second, the outcome of the safe gamble is revealed. In addition, the subject sees that outcome of the chosen gamble. Thus, a subject who chooses the risky gamble will know the outcome only of the risky gamble in the first condition and both gambles in the second, and the subject who chooses the safe gamble will know the outcome of both gambles in the first condition and only the safe gamble in the second. Subjects most frequently chose the gamble that was to be revealed in that condition, demonstrating a preference to not know the outcome of the foregone alternative. Hence, the subjects chose actions that minimize regret, independent of their decisions to play "risky" or "safe." But in this case, regret is avoided by *suppressing information about the foregone alternative*, and not by choosing regret-minimizing actions. Nevertheless, the implications to the insurance story are clear. One could experience regret by either not purchasing enough insurance that one ultimately needs (risk-averse behavior avoids this condition), or by purchasing more insurance than is ultimately needed (a compensatory reduction in insurance coverage is risk seeking).

Empirical Justification of Regret Theory

There exists a significant body of evidence to suggest that Regret Theory can explain deviations from EUT that have been observed in both the laboratory and in the field. Loomes and Sugden (1982) show how Kahneman and Tversky's examples of preference inconsistencies and intransitivities that were explained by Prospect Theory (Kahneman and Tversky, 1979) can also be explained by Regret Theory. Bell (1982) illustrates how Regret Theory can help explain simultaneous preferences for insurance and gambling. Experiments to test the specific impact of regret (as opposed to other drivers such as disappointment) on preferences were conducted by Loomes and Sugden (1987) with weak results, but later replication and refinement by Loomes (1988) demonstrated that the influence of regret is, in fact, quite strong. Loomes, Starmer, and Sugden (1992) subsequently confirmed experimentally that regret is a significant factor when preferences violate stochastic dominance. And while Starmer and Sugden (1993) showed that effects related to the presentation of the lotteries in the earlier experiments might have strengthened the earlier results, the impact of regret is only marginally less significant than previously thought. Furthermore, these qualifications still do not apply to our particular model where optimal decisions are continuous within contiguous states of the world.

Attempts outside of the laboratory to validate the importance of regret in decision making are more encouraging. For example, Connolly and Reb (2003), through work on omission bias, found that decisions about whether to get a vaccination tended to be driven by the avoidance of regret. These results are interesting in our context because the anticipated regret works in two directions: some subjects did not get shots because they wanted to avoid risks from the vaccine, while others actively sought out vaccination so they could avoid the disease itself. This decision is similar to the insurance case, in which an individual must balance regret from buying too little insurance with regret from buying too much. Baron and Hershey (1988) did find consistent evidence that the *ex post* evaluation of the quality of a decision is influenced by the favorability of the final outcomes. In other words, a decision after the fact is more likely to be considered "good" if the final outcome was good. The authors report a weakly positive change in the *ex post* quality of decisions when the attention of subjects was focused on these relative outcomes of the alternatives. However, in one experiment, the authors did find significant evidence that a decision is more likely to be considered good, if the difference between the outcomes of the chosen and foregone alternatives is high. Although this result could constitute "rejoicing" as well as regret, it is still consistent with the idea of anticipating *ex post* suboptimal outcomes.

Of course, even if the experimental evidence on regret is inconclusive, it does not mean that regret is not a factor in decision making (Baron, 2000, p. 265). As we develop our model of regret-induced decision making, we will show below that regret can perturb optimal decisions in one direction in some situations, but in other directions for others.

THE INSURANCE MODEL

We now turn our attention to the application of the RTEU function to optimal insurance purchase decisions. Suppose an individual is endowed with an initial level of wealth $w_0 \geq 0$ and faces a monetary, random loss X, which is characterized by a cumulative distribution function $F : [0, w_0] \rightarrow \mathbb{R}$ with $F(0) = 0$ and $F(w_0) = 1$.[4] An insurance company offers a menu of indemnity insurance contracts to the individual for premiums that are set equal to the expected indemnity plus a proportional loading factor $\lambda \geq 0$. This pricing behavior can be justified economically by assuming that the insurer is risk neutral in a perfectly competitive insurance market with proportional transaction costs but no entry costs. We exclude any informational asymmetries that would give rise to moral hazard or adverse selection problems. The individual's decision making is assumed to be representable by RTEU maximization subject to a Bernoulli utility function $u : \mathbb{R}_+ \rightarrow \mathbb{R}$, which, due to regret aversion, exhibits the structure given by Equation (2), i.e.,

$$u(w) = v(w) - k \cdot g(v(w^{\max}) - v(w))$$

for all levels of final wealth $w \geq 0$. (The definitions and conditions for these functions were presented in the "The RTEU Function" section.)

In the next two subsections, we examine how regret impacts the individual's insurance purchasing decision if the insurer offers a coinsurance and a deductible policy. In all cases, when we refer to the amount of insurance, we refer to the coinsurance rate or deductible level and not an upper limit on coverage.

Coinsurance Policy

Suppose an insurer offers a coinsurance contract with coinsurance rates $\alpha \in [0, 1]$. The indemnity schedule $I : [0, w_0] \rightarrow \mathbb{R}$ is given by

$$I(x) = \alpha x$$

for all realized losses $x \in \mathbb{R}_+$ of the random variable X and premiums are equal to

$$P(\alpha) = (1 + \lambda)E[I(X)] = (1 + \lambda)\alpha E[X], \tag{4}$$

where $\lambda \geq 0$ is the proportional loading factor.

The individual chooses a coinsurance rate α to maximize RTEU of final wealth. As a function of α, final wealth is

$$w(\alpha) = w_0 - (1 + \lambda)\alpha E[X] - (1 - \alpha)X.$$

To determine the optimal insurance purchasing decision, we first deduce the *ex post* optimal level of final wealth $w^{\max}$.

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[4] Our results also hold for the case where there is a strictly positive probability that no loss occurs.

Lemma 1: *The* ex post *optimal insurance purchasing decision is full insurance* ($\alpha = 1$) *if the realized loss x exceeds* $(1 + \lambda)\, E[X]$ *and no insurance* ($\alpha = 0$) *otherwise. The* ex post *optimal level of final wealth is* $w^{\max} = w_0 - \min(x, (1 + \lambda)\, E[X])$.

These *ex post* choices are interpreted as the amount of insurance the individual would have purchased had the actual amount of the loss been known in advance. If the loss were less than the premium for full insurance, then the individual would have done better by paying for the loss *instead* of the premium. Similarly, if the amount of the loss were greater than the premium for full insurance, the individual is better off purchasing full coverage for that loss.

Mossin (1968) has shown that a risk-averse individual who is not regret averse would buy full insurance ($\alpha^* = 1$) if the contract were fairly priced. Partial insurance may be indicated either by a positive loading factor ($\lambda > 0$), or by moral hazard (Holmstrom, 1979) or adverse selection (Rothschild and Stiglitz, 1976). We now establish another reason why individuals would demand partial insurance in the absence of such information asymmetries.

Proposition 1: *If a coinsurance contract is offered, a regret-averse individual purchases partial insurance* ($\alpha^* < 1$) *even at a fair price* ($\lambda = 0$).

Consider a regret-averse individual who purchases full insurance under a coinsurance contract that is fairly priced ($\lambda = 0$). If $x > E[X]$, the individual experiences no regret—he or she has made the *ex post* optimal decision. Otherwise, the individual experiences a lot of regret from buying any insurance at all. By reducing the coinsurance rate, the individual experiences regret in all states of the world—more regret for $x > E[X]$ and less regret otherwise. Because g is convex, this adjustment reduces the expected disutility of regret. Therefore, full insurance cannot be optimal.

Let $\alpha_k^*(\lambda)$ denote the optimal coinsurance rate for a given loading factor $\lambda \geq 0$ and regret coefficient $k \geq 0$. We have already shown that for $k > 0$, $\alpha_k^*(\lambda) < 1$ for all $\lambda \geq 0$. In the following proposition, we show how the optimal coinsurance level $\alpha_k^*(0)$ responses to changes in the regret coefficient k, if the contract is fairly priced.

Proposition 2: *If a fairly priced coinsurance contract is offered, an individual who has a higher regret coefficient k purchases less insurance coverage than the individual who has a lower regret coefficient (i.e.,* $\frac{d\alpha_k^*(0)}{dk} < 0$).

We know from Mossin (1968) that there exists some loading factor $\bar{\lambda} > 0$ above which the individual will not purchase insurance (i.e., $\alpha_0^*(\bar{\lambda}) = 0$). In the next proposition, we show that a regret-averse individual purchases some insurance at this loading factor and that more regret aversion leads to a higher level of insurance.

Proposition 3: *If a coinsurance contract is offered with a loading factor*

$$\bar{\lambda} = \frac{\mathrm{Cov}[v'(w_0 - X), X]}{E[v'(w_0 - X)]E[X]} > 0,$$

then a regret-averse individual ($k > 0$) *will purchase some nonzero amount of insurance* ($\alpha_k^*(\bar{\lambda}) > 0$) *while a non-regret-averse individual* ($k = 0$) *would not buy any insurance*

$(\alpha_0^*(\bar{\lambda}) = 0)$. *At this loading factor, the optimal coinsurance rate is increasing (i.e.,* $\frac{d\alpha_k^*(\bar{\lambda})}{dk} > 0$) i *n the regret coefficient k.*

The intuition behind this result parallels that of the fairly priced contract. The expected disutility of regret can be reduced by purchasing some amount of insurance.

So far we have derived the following results:

$$\begin{aligned} \alpha_k^*(\lambda) &< 1 \quad \text{for all } k > 0, \lambda \geq 0, \\ \frac{d\alpha_k^*(0)}{dk} &< 0 \quad \text{for all } k > 0, \\ \alpha_k^*(\bar{\lambda}) &> 0 \quad \text{for all } k > 0, \\ \frac{d\alpha_k^*(\bar{\lambda})}{dk} &> 0 \quad \text{for all } k > 0. \end{aligned}$$

These results mean that for $\lambda = 0$, regret induces an individual to buy partial insurance instead of full insurance, and at $\lambda = \bar{\lambda}$, regret induces an individual to purchase partial insurance instead of no insurance. Finally, there exists a loading factor for which regret has no effect on the optimal amount of insurance. This loading factor reflects the point at which the impact of regret switches from buying less insurance to more insurance relative to the optimal amount of an individual who is not regret averse.

Proposition 4: *There exists a loading factor* $\hat{\lambda} \in (0, \bar{\lambda})$ *such that* $\alpha_k^*(\hat{\lambda}) = \alpha_0^*(\hat{\lambda})$ *for all* $k \geq 0$.

We illustrate the effects of regret on the optimal coinsurance rate in Figure 1.

Aside from the endpoints of the $\alpha_0^*(\lambda)$ plot, we do not know the exact *shape* of either curve in the interior. In EUT-based insurance demand models, the slope and concavity of $\alpha_0^*(\lambda)$ are driven by income and substitution effects, and this characteristic remains for the RTEU model. However, we see that for low loading factors, adding regret induces the purchase of less insurance, while high loading factors imply that regret leads to purchasing more insurance. These results do *not* imply necessarily that individuals purchase more insurance at higher loading factors. Rather, the perturbations in the optimal insurance demand should be thought of as the amount of insurance demanded by the regret-averse individual *relative* to that of the straight expected utility maximizer. Nor could these results be replicated by altering the risk aversion of a single attribute expected utility function, since the EUT predictions at the endpoints (full insurance or no insurance) hold for a risk-averse individual with any concave utility function.

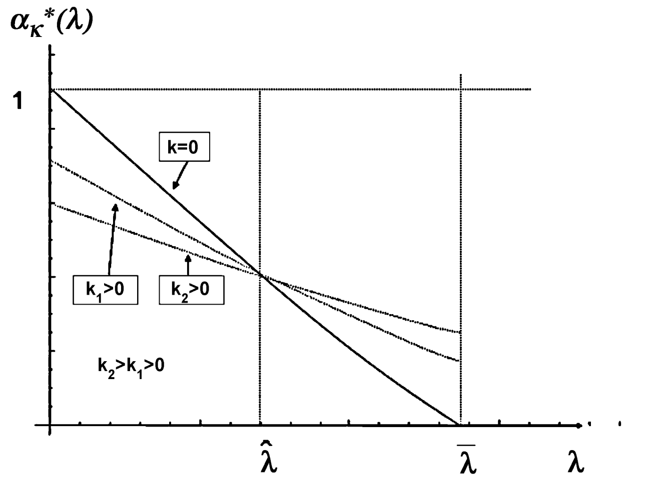
Deductible Policy

We now turn to insurance policies with deductibles. Suppose an insurer offered a set of deductible contracts with deductible levels $D \in [0, w_0]$. The indemnity schedule is thus

$$I(x) = \max(x - D, 0) = (x - D)^+$$

FIGURE 1

Plots the Optimal Coinsurance Rate Against Loading Factors. For $k = 0$, an Individual Demands Full Insurance for $\lambda = 0$ and No Insurance for $\lambda = \bar{\lambda}$. For $k > 0$, the Individual Demands Partial Insurance at Both $\lambda = 0$ and $\lambda = \bar{\lambda}$. The Amount of the Adjustment Increases as the Regret Coefficient k Increases



and the premium is given by

$$P(D) = (1+\lambda)E[(X-D)^+].$$

Differentiation yields

$$P'(D) = -(1+\lambda)(1-F(D)). \tag{5}$$

The individual chooses a deductible level D to maximize RTEU of final wealth. As a function of the deductible level D, final wealth is

$$\begin{aligned} w(D) &= w_0 - (1+\lambda)E[(X-D)^+] - X + (X-D)^+ \\ &= w_0 - (1+\lambda)E[(X-D)^+] - \min(X, D). \end{aligned}$$

In the following lemma, we derive the *ex post* optimal final level of wealth, $w^{\max}$.

Lemma 2: *It is* ex post *optimal to buy a deductible level $\bar{D}(\lambda)$ if the realized loss x exceeds $(1+\lambda)E[(X-\bar{D}(\lambda))^+] + \bar{D}(\lambda)$, and to buy no insurance otherwise. $\bar{D}(\lambda)$ is the $\frac{\lambda}{1+\lambda}$th quantile of the loss distribution function F, i.e., $\bar{D}(\lambda) = F^{-1}(\frac{\lambda}{1+\lambda})$ where F^{-1} is the generalized inverse of F. The* ex post *optimal level of final wealth $w^{\max}$ is given by*

$$w^{\max} = w_0 - \min(x, (1+\lambda)E[(X-\bar{D}(\lambda))^+] + \bar{D}(\lambda)).$$

The following proposition shows that an individual who is both risk averse and regret averse will demand partial insurance under a deductible policy, even if the contract is fairly priced ($\lambda = 0$).

Proposition 5: *If a deductible insurance contract is offered, a regret-averse individual purchases less than full insurance* ($D^* > 0$) *even at a fair price* ($\lambda = 0$).

Now let $D_k^*(\lambda)$ denote the optimal deductible level for a given loading factor $\lambda \geq 0$ and regret coefficient $k \geq 0$. Analogous to the coinsurance case, we will show how the optimal deductible level $D_k^*(0)$ responds to changes in the regret coefficient k if the contract is fairly priced.

Proposition 6: *If a fairly priced* ($\lambda = 0$) *deductible contract is offered, an individual with a higher regret coefficient k purchases less insurance coverage relative to an individual with a lower regret coefficient k (i.e.,* $\frac{dD_k^*(0)}{dk} > 0$).

Next, we note that there exists some loading factor $\bar{\lambda}$ above which an individual is risk averse, but not regret averse, will purchase no insurance. However, a regret-averse individual will purchase some insurance priced with this loading factor $\bar{\lambda}$ and more regret aversion leads to a higher level of insurance, i.e., to a lower deductible level.

Proposition 7: *If a deductible contract is offered with a loading factor*

$$\bar{\lambda} = \frac{v'(0)}{E[v'(w_0 - X)]} - 1 > 0,$$

then a regret-averse individual ($k > 0$) *purchases some nonzero amount of insurance* ($D_k^*(\bar{\lambda}) < w_0$) *while a non-regret-averse individual* ($k = 0$) *will purchase no insurance* ($D_0^*(\bar{\lambda}) = w_0$). *At this loading factor, the optimal deductible level is decreasing (i.e.,* $\frac{dD_k^*(\bar{\lambda})}{dk} < 0$) *in the regret coefficient k.*

We have thus derived results that are analogous to coinsurance, namely

$$\begin{aligned} D_k^*(\lambda) &> 0 \quad && \text{for all } k > 0, \lambda \geq 0, \\ \frac{dD_k^*(0)}{dk} &> 0 && \text{for all } k > 0, \\ D_k^*(\bar{\lambda}) &< w_0 && \text{for all } k > 0, \\ \frac{dD_k^*(\bar{\lambda})}{dk} &< 0 && \text{for all } k > 0. \end{aligned}$$

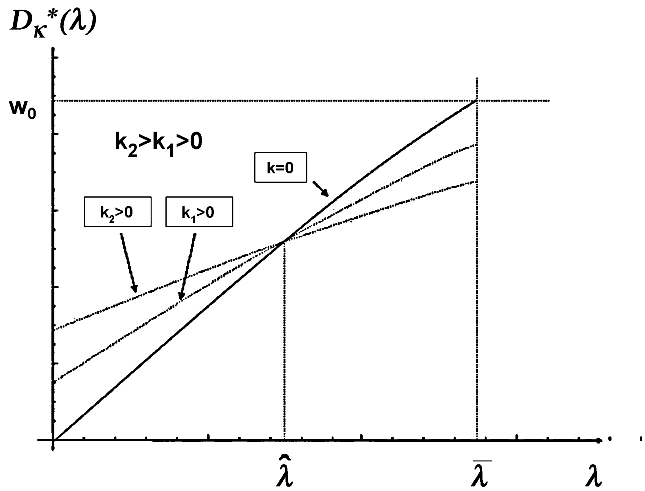
Finally, we show that there exists a loading factor for which regret has no effect on the insurance choice and that this loading factor is the point at which the impact of regret switches from less insurance to more insurance.

Proposition 8: *There exists a loading factor* $\hat{\lambda} \in (0, \bar{\lambda})$ *such that* $D_k^*(\hat{\lambda}) = D_0^*(\hat{\lambda})$ *for all* $k \geq 0$.

Figure 2 illustrates the impact of regret on the optimal deductible level, which exactly parallels that of the coinsurance case.

FIGURE 2

Plots the Optimal Deductible Level Against Loading Factors. For $k = 0$, an Individual Demands Full Insurance for $\lambda = 0$ and No Insurance for $\lambda = \bar{\lambda}$. For $k > 0$, the Individual Demands Partial Insurance at Both $\lambda = 0$ and $\lambda = \bar{\lambda}$. The Amount of the Adjustment Increases as the Regret Coefficient k Increases



DISCUSSION

Preferences for Low Deductibles

Our results establish conditions under which regret can help explain observed preferences for low deductibles; the regret-averse individual will buy more insurance than the non-regret-averse individual as long as the loading factor is sufficiently high. The existence of the preference for low deductibles is well known. One of the first empirical studies of the low deductible phenomenon was conducted by Pashigian, Schkade, and Menefee (1966), who derived ranges into which the insurance premia of an expected utility maximizing individual (assuming a quadratic utility function) must fall. Their results showed that out of a sample of more than 4.8 million insured drivers in 1962, about 53.8 percent chose the lowest deductible and another 45.7 percent chose the next lowest. Yet, none of the premia paid by those drivers fell within the range that would be consistent with maximizing expected utility; only the sliver choosing the highest deductibles fell within that range. In the late 1970s, a proposal in Pennsylvania to place a minimum deductible of \$100 on automobile insurance policies was ultimately rescinded after public outcry, even though such legislation could have saved consumers millions of dollars each year (Cummins et al., 1978; Kunreuther, 2000). More recently, Grace et al. (2003) summarized preferences for low deductibles for homeowners' insurance in New York and Florida for catastrophic losses. Preferences for low deductibles have also been indicated experimentally. Johnson et al. (1993) asked subjects to choose between two hypothetical insurance policies. The first policy was structured with a deductible. The second policy was actuarially identical to the first, but had no deductible, a higher premium and a rebate to the customer in the event no claim was filed. More than two-thirds of the subjects preferred the rebate option to the deductible option, suggesting a disproportionate aversion to covering the deductible portion of the total loss. Slovic et al. (1977) conducted experiments of both the "ball in urn" and "simulated business insurance situation" variety that suggest that individuals are more likely to insure against high-probability, low-impact events than against low-probability, high-impact ones of common expected value. These two results demonstrate the preference for individuals to pay in advance in order to avoid future losses, since policies with lower deductibles or higher coinsurance rates offer higher amounts of reimbursement once those losses are accrued.

In considering the impact of our results, we did not consider another possible explanation for preferences for low deductibles. We may observe these preferences because a large number of insurance customers choose the lowest deductible contracts *that are presented to them*. It is possible that we see clustering at the low end of the scale, because the highest deductible contract is just too high.

CONCLUSION AND FUTURE RESEARCH

We have shown that individuals with regret-theoretical preferences will "hedge their bets" when making insurance decisions for coinsurance contracts, with equivalent results for deductible contracts. Regardless of the loading factor, the anticipation of disutility from regret creates a mitigating effect on the insurance purchase decision. When EUT predicts a high optimal level of insurance, the RTEU model compensates for the fact that there will be some states of the world for which no insurance is optimal *ex post*. Similarly, if EUT were to predict a low level of insurance, the present model predicts a higher level to take into account those states of the world for which more insurance is optimal *ex post*. No matter what insurance decision is made, the individual is adjusting for the possibility of either buying too little insurance if a large loss is incurred, or too much insurance that is never used. Thus, anticipation of regret should prevent an individual from making extreme decisions. These results cannot be explained by risk-aversion alone, since any risk-averse individual will buy full insurance when the insurance is fairly price (Mossin, 1968; Schlesinger, 1981, 2000).

These findings have significant implications for sellers of insurance and for regulators. One possible path for future research is to examine the structure of the optimal insurance policy when customers have regret-theoretical preferences. These preferences could induce an insurance company to adjust loading factors or the menu of insurance levels to compensate for the mitigation effect. A regulator, however, may be concerned with incentives for insurance companies to manipulate regret through marketing activities. If marketers could induce an individuals preferences to be more regret-theoretical and, as a result of such a shift the optimal loading factor goes up, then the marketer is essentially raising the price of and the demand for insurance simultaneously. But if the loading factor were too low, inducing regret might lead individuals to purchase too little insurance, placing the customers at a greater risk of experiencing an unmanageable loss. Our present model opens the door for additional experimental and empirical work to better understand how tolerances of risk and regret influence actual insurance decisions. We also see the potential for expanding this analytical approach to the study of regret to areas outside of insurance. Any situation in which individuals "hedge their bets" as a risk-management strategy would be targetable. If we did not continue this research, we would certainly regret it.

APPENDIX

Proof of Lemma 1: If a loss of severity x occurs, then the individual's final level of wealth is

$$\begin{aligned} w(\alpha) &= w_0 - (1+\lambda)\alpha E[X] - x + \alpha x \\ &= w_0 - x - \alpha[(1+\lambda)E(X) - x]. \end{aligned}$$

If $x < (1 + \lambda)\, E[X]$, then $w(\alpha)$ is maximized at $\alpha = 0$. Otherwise, $w(\alpha)$ is maximized at $\alpha = 1$. Therefore, the optimal *ex post* level of final wealth $w^{\max}$ for a realized loss of severity x is given by

$$w^{\max} = \begin{cases} w(0) = w_0 - x, & \text{if } x < (1+\lambda)E[X], \\ w(1) = w_0 - (1+\lambda)E[X], & \text{if } x \geq (1+\lambda)E[X]. \end{cases}$$

Q.E.D.

Proof of Proposition 1: The individual's optimization problem is given by

$$\max_{\alpha\in[0,1]} E[u(w(\alpha))] = E[v(w(\alpha))] - kE[g(v(w^{\max}) - v(w(\alpha)))],$$

where

$$\begin{aligned} w(\alpha) &= w_0 - (1+\lambda)\alpha E[X] - X + \alpha X, \\ w^{\max} &= w_0 - \min(X, (1+\lambda)E[X]). \end{aligned}$$

The first derivative of RTEU with respect to α is given by

$$\begin{aligned} \frac{dE[u(w(\alpha))]}{d\alpha} &= \frac{dE[v(w(\alpha))]}{d\alpha} + kE[v'(w(\alpha))(X-(1+\lambda)E[X])g'(v(w^{\max}) - v(w(\alpha)))] \\ &= \frac{dE[v(w(\alpha))]}{d\alpha} \\ &\quad + k\int_0^{(1+\lambda)E[X]} v'(w(\alpha))(x-(1+\lambda)E[X])g'(v(w(0)) - v(w(\alpha)))\, dF(x) \\ &\quad + k\int_{(1+\lambda)E[X]}^{w_0} v'(w(\alpha))(x-(1+\lambda)E[X])g'(v(w(1)) - v(w(\alpha)))\, dF(x). \end{aligned} \tag{A1}$$

The second derivative is

$$\frac{d^2E[u(w(\alpha))]}{d\alpha^2} = \frac{d^2E[u(w(\alpha))]}{d\alpha^2} + kE[v''(w(\alpha))(X-(1+\lambda)E[X])^2 g'(v(w^{\max}) - v(w(\alpha)))]$$
$$- kE[[v'(w(\alpha))]^2 (X-(1+\lambda)E[X])^2 g''(v(w^{\max}) - v(w(\alpha)))]. \tag{A2}$$

Since $g' > 0$ and $g'' > 0$, the second derivative given by Equation (A2) is strictly negative for all values of α. This ensures that RTEU is a *globally* concave function in α and hence any solution α^* of the first-order condition (FOC)

$$\frac{dE[u(w(\alpha))]}{d\alpha} = 0 \tag{A3}$$

is a global maximum. Evaluating the first derivative (A1) for $k > 0$ at the full insurance point $\alpha = 1$ yields

$$\begin{aligned}
\left.\frac{dE[u(w(\alpha))]}{d\alpha}\right|_{\alpha=1} &= \left.\frac{dE[v(w(\alpha))]}{d\alpha}\right|_{\alpha=1} + kv'(w(1)) \\
&\quad \times \int_0^{(1+\lambda)E[X]} (x-(1+\lambda)E[X]) g'(v(w(0)) - v(w(1)))\, dF(x) \\
&\quad + kv'(w(1))g'(0) \int_{(1+\lambda)E[X]}^{w_0} (x-(1+\lambda)E[X])\, dF(x) \\
&< \left.\frac{dE[v(w(\alpha))]}{d\alpha}\right|_{\alpha=1} - kv'(w(1))g'(0)\lambda E[X] \\
&= -\lambda E[X]v'(w(1))(1+kg'(0)) \\
&\leq 0.
\end{aligned}$$

Hence,

$$\left.\frac{dE[u(w(\alpha))]}{d\alpha}\right|_{\alpha=1} < 0$$

for all $k > 0$ and $\lambda \geq 0$. This implies that at the full insurance level, the individual can increase RTEU by reducing the coinsurance rate. Therefore, the individual chooses a coinsurance rate $\alpha^* < 1$ even if the contract is actuarially fairly priced ($\lambda = 0$). Q.E.D.

Proof of Proposition 2: We need to show that $\frac{d\alpha_k^*(0)}{dk} < 0$. Applying the total differential to the FOC (Equation (A3)) at the optimal coinsurance rate $\alpha_k^*(\lambda)$ leads to

$$\left.\frac{\partial^2 E[u(w(\alpha))]}{\partial\alpha^2}\right|_{\alpha=\alpha_k^*(\lambda)} \cdot d\alpha_k^*(\lambda) + \left.\frac{\partial^2 E[u(w(\alpha))]}{\partial\alpha\partial k}\right|_{\alpha=\alpha_k^*(\lambda)} \cdot dk = 0.$$

Hence

$$\frac{d\alpha_k^*(\lambda)}{dk} = -\frac{\left.\dfrac{\partial^2 E[u(w(\alpha))]}{\partial\alpha\partial k}\right|_{\alpha=\alpha_k^*(\lambda)}}{\left.\dfrac{\partial^2 E[u(w(\alpha))]}{\partial\alpha^2}\right|_{\alpha=\alpha_k^*(\lambda)}}. \tag{A4}$$

From (A2), we know that RTEU is globally concave in α, so $\frac{\partial^2 E[u(w(\alpha))]}{\partial\alpha\partial k}|_{\alpha=\alpha_k^*(\lambda)} < 0$. Therefore,

$$\text{sign}\left(\frac{d\alpha_k^*(\lambda)}{dk}\right) = \text{sign}\left(\left.\frac{\partial^2 E[u(w(\alpha))]}{\partial\alpha\partial k}\right|_{\alpha=\alpha_k^*(\lambda)}\right).$$

Differentiating the first derivative (A1) with respect to k yields

$$\frac{\partial^2 E[u(w(\alpha))]}{\partial\alpha\partial k} = E[v'(w(\alpha))(X-(1+\lambda)E[X])g'(v(w^{\max}) - v(w(\alpha)))].$$

Substituting this expression back into Equation (A1) leads to

$$\frac{dE[u(w(\alpha))]}{d\alpha} = \frac{dE[v(w(\alpha))]}{d\alpha} + k\frac{\partial^2 E[u(w(\alpha))]}{\partial\alpha\partial k}.$$

At $\alpha = \alpha_k^*(\lambda)$, the FOC (A3) implies

$$\left.\frac{dE[v(w(\alpha))]}{d\alpha}\right|_{\alpha=\alpha_k^*(\lambda)} = -k\left.\frac{\partial^2 E[u(w(\alpha))]}{\partial\alpha\partial k}\right|_{\alpha=\alpha_k^*(\lambda)},$$

so

$$\text{sign}\left(\left.\frac{dE[v(w(\alpha))]}{d\alpha}\right|_{\alpha=\alpha_k^*(\lambda)}\right) = -\text{sign}\left(\left.\frac{\partial^2 E[u(w(\alpha))]}{\partial\alpha\partial k}\right|_{\alpha=\alpha_k^*(\lambda)}\right)$$

and thus

$$\text{sign}\left(\frac{d\alpha_k^*(\lambda)}{dk}\right) = -\text{sign}\left(\left.\frac{dE[v(w(\alpha))]}{d\alpha}\right|_{\alpha=\alpha_k^*(\lambda)}\right). \tag{A5}$$

In Proposition 1, we showed that $\alpha_k^*(0) < 1$ for fairly priced contracts for all $k > 0$. From Mossin (1968), we know that it is optimal for a purely risk-averse individual with utility function v to buy full insurance if the contract is fairly priced. Global concavity of EU with respect to α then implies

$$\left.\frac{dE[v(w(\alpha))]}{d\alpha}\right|_{\alpha=\alpha_k^*(0)} > 0.$$

From (A5) we conclude that

$$\frac{d\alpha_k^*(0)}{dk} < 0.$$

So for fairly priced insurance, more regret (with respect to k) induces an individual to buy less insurance. Q.E.D.

Proof of Proposition 3: For $k = 0$,

$$\left.\frac{dE[u(w(\alpha))]}{d\alpha}\right|_{\alpha=0} = E[v'(w(0))(X - (1+\lambda)E[X])].$$

We find the loading factor $\bar{\lambda}$, at which a risk-averse individual who does not consider regret buys no insurance coverage, by solving

$$E[v'(w(0))(X - (1+\bar{\lambda})E[X])] = 0, \tag{A6}$$

which yields

$$\bar{\lambda} = \frac{\text{Cov}[v'(w_0 - X), X]}{E[v'(w_0 - X)]E[X]}.$$

For $k > 0$, evaluating the first derivative (A1) at $\alpha = 0$ yields

$$\begin{aligned}
\left.\frac{dE[u(w(\alpha))]}{d\alpha}\right|_{\alpha=0} &= \left.\frac{dE[v(w(\alpha))]}{d\alpha}\right|_{\alpha=0} + kg'(0)\int_0^{(1+\lambda)E[X]} v'(w(0))(x - (1+\lambda)E[X])\,dF(x) \\
&\quad + k\int_{(1+\lambda)E[X]}^{w_0} v'(w(0))(x - (1+\lambda)E[X])g'(v(w(1)) - v(w(0)))\,dF(x) \\
&> \left.\frac{dE[v(w(\alpha))]}{d\alpha}\right|_{\alpha=0} + kg'(0)E[v'(w(0))(X - (1+\lambda)E[X])] \\
&= E[v'(w(0))(X - (1+\lambda)E[X])](1 + kg'(0)).
\end{aligned}$$

If the contract is priced at the loading factor $\bar{\lambda}$, Equation (A6) implies

$$\left.\frac{dE[u(w(\alpha))]}{d\alpha}\right|_{\alpha=0} > 0$$

for all $k > 0$. We thus conclude that $\alpha_k^*(\bar{\lambda}) > 0$ for all $k > 0$.

As the purely risk-averse individual would not buy any insurance at the loading factor $\bar{\lambda}$, i.e., $\alpha_0^*(\bar{\lambda}) = 0$, global concavity of EU implies

$$\left.\frac{dE[v(w(\alpha))]}{d\alpha}\right|_{\alpha=\alpha_k^*(\bar{\lambda})} < 0$$

for all $k > 0$. From (A5), we conclude that

$$\frac{d\alpha_k^*(\bar{\lambda})}{dk} > 0. \qquad \text{Q.E.D.}$$

Proof of Proposition 4: For an arbitrarily fixed $k > 0$, define $\Delta_k(\lambda) = \alpha_0^*(\lambda) - \alpha_k^*(\lambda)$. We have $\Delta_k(0) > 0$ from Proposition 1 and $\Delta_k(\bar{\lambda}) < 0$ from Proposition 3. Since $\Delta_k(\lambda)$ is a continuous function in λ, the Intermediate Value Theorem implies that there must exist a $\hat{\lambda}_k$ with $0 < \hat{\lambda}_k < \bar{\lambda}$ such that

$$\Delta_k(\hat{\lambda}_k) = 0,$$

so

$$\alpha_k^*(\hat{\lambda}_k) = \alpha_0^*(\hat{\lambda}_k).$$

We next show that this intermediate loading factor $\hat{\lambda}_k$ is independent of k. As $\alpha_k^*(\hat{\lambda}_k)$ is an inner solution, $\hat{\lambda}_k$ is determined by the FOC

$$\left.\frac{dE[v(w(\alpha))]}{d\alpha}\right|_{\alpha=\alpha_0^*(\hat{\lambda}_k)} = 0$$

for $k = 0$ and

$$\left.\frac{dE[u(w(\alpha))]}{d\alpha}\right|_{\alpha=\alpha_k^*(\hat{\lambda}_k)} = 0$$

for $k > 0$. As $\alpha_k^*(\hat{\lambda}_k) = \alpha_0^*(\hat{\lambda}_k)$ those two conditions reduce to the following:

$$E\left[v'\left(w\left(\alpha_0^*(\hat{\lambda}_k)\right)\right)(X - (1 + \hat{\lambda}_k)E[X])g'\left(v(w^{\max}) - v\left(w\left(\alpha_0^*(\hat{\lambda}_k)\right)\right)\right)\right] = 0$$

according to (A1). As this condition is independent of k, $\hat{\lambda}_k = \hat{\lambda}$ for all $k > 0$ and thus $\alpha_k^*(\hat{\lambda}) = \alpha_0^*(\hat{\lambda})$ for all $k > 0$. Q.E.D.

Proof of Lemma 2: It is never *ex post* optimal for the individual to have purchased insurance with a deductible level above the realized loss x, since it would be more costly than not buying any insurance at all. In this case, the final level of wealth is $w_0 - x$. The optimal *ex post* deductible level $\bar{D}(\lambda)$ below the realized loss x maximizes

$$w(D) = w_0 - (1 + \lambda)E[(X - D)^+] - D.$$

The first- and second-derivatives are

$$w'(D) = (1+\lambda)(1-F(D)) - 1,$$
$$w''(D) = -(1+\lambda)f(D) < 0,$$

where $f(\cdot)$ is the density function to $F(\cdot)$.

$w'(D) = 0$ if and only if

$$F(D) = \frac{\lambda}{1+\lambda},$$

so

$$\bar{D}(\lambda) = F^{-1}\left(\frac{\lambda}{1+\lambda}\right).$$

The *ex post* optimal level of final wealth is

$$w^{\max} = \begin{cases} w_0 - x, & \text{if } x < P(\bar{D}(\lambda)) + \bar{D}(\lambda), \\ w_0 - P(\bar{D}(\lambda)) - \bar{D}(\lambda), & \text{if } x \geq P(\bar{D}(\lambda)) + \bar{D}(\lambda). \end{cases} \tag{A7}$$

Proof of Proposition 5: The individual solves the following optimization problem

$$\max_{D\in[0,w_0]} E[u(w(D))] = \int_0^D \big(v\big(w_0 - P(D) - x\big) - kg\big(v(w^{\max}) - v(w_0 - P(D) - x)\big)\big)\, dF(x)$$
$$+ \int_D^{w_0} \big(v\big(w_0 - P(D) - D\big) - kg\big(v(w^{\max}) - v(w_0 - P(D) - D)\big)\big)\, dF(x).$$

The first derivative yields

$$\frac{dE[u(w(D))]}{dD}$$
$$= -P'(D)\int_0^D v'(w_0 - P(D) - x)\big(1 + kg'\big(v(w^{\max}) - v(w_0 - P(D) - x)\big)\big)\, dF(x)$$
$$-(P'(D)+1)v'(w_0 - P(D) - D)\int_D^{w_0} \big(1 + kg'\big(v(w^{\max}) - v(w_0 - P(D) - D)\big)\big)\, dF(x), \tag{A8}$$

and the second derivative

$$
\begin{aligned}
&\frac{d^2E[u(w(D))]}{dD^2} \\
&\quad = -P''(D)\int_0^D v'(w_0 - P(D) - x)\big(1 + kg'\big(v(w^{\max}) - v(w_0 - P(D) - x)\big)\big)\,dF(x) \\
&\qquad + P'^2(D)\int_0^D v''(w_0 - P(D) - x)\big(1 + kg'\big(v(w^{\max}) - v(w_0 - P(D) - x)\big)\big)\,dF(x) \\
&\qquad - P'^2(D)\int_0^D v'^2(w_0 - P(D) - x)kg''\big(v(w^{\max}) - v(w_0 - P(D) - x)\big)\,dF(x) \\
&\qquad - P''(D)v'(w_0 - P(D) - D)\int_D^{w_0}\big(1 + kg'\big(v(w^{\max}) - v(w_0 - P(D) - D)\big)\big)\,dF(x) \\
&\qquad + (P'(D)+1)^2v''(w_0 - P(D) - D)\int_D^{w_0}\big(1 + kg'\big(v(w^{\max}) - v(w_0 - P(D) - D)\big)\big)\,dF(x) \\
&\qquad - (P'(D)+1)^2v'(w_0 - P(D) - D)\int_D^{w_0} kg''\big(v(w^{\max}) - v(w_0 - P(D) - D)\big)\,dF(x) \\
&\qquad + f(D)v'(w_0 - P(D) - D)\big(1 + kg'\big(v(w^{\max}(D)) - v(w_0 - P(D) - D)\big)\big), \qquad \text{(A9)}
\end{aligned}
$$

where from Equation (A7), $w^{\max}(D) = w_0 - \min(D, (1+\lambda)E[(X - \bar{D}(\lambda))^+] + \bar{D}(\lambda))$. At $D = 0$, the first derivative is

$$
\left.\frac{dE[u(w(D))]}{dD}\right|_{D=0} = -(P'(0)+1)v'(w_0 - P(0)) \times \int_0^{w_0}\big(1 + kg'\big(v(w^{\max}) - v(w_0 - P(0))\big)\big)\,dF(x).
$$

From (5) we get

$$
\left.\frac{dE[u(w(D))]}{dD}\right|_{D=0} = \lambda v'(w_0 - P(0)) \times \int_0^{w_0}\big(1 + kg'\big(v\big(w_L^{\max}\big) - v(w_0 - P(0))\big)\big)\,dF(x), \qquad \text{(A10)}
$$

which is positive if $\lambda > 0$. This implies that $D^* > 0$ for all $\lambda > 0$.

If $\lambda = 0$ then

$$\left.\frac{dE[u(w(D))]}{dD}\right|_{D=0} = 0.$$

To determine whether $D > 0$ is optimal in this situation, we examine whether RTEU is a convex or concave function in D at $D = 0$.[5] For $\lambda = 0$, we derive from (5) and (A8)

$$\begin{aligned}\frac{dE[u(w(D))]}{dD}\\ &= (1 - F(D)) \int_0^D v'(w_0 - P(D) - x)\big(1 + kg'\big(v(w^{\max}) - v(w_0 - P(D) - x)\big)\big)\, dF(x)\\ &\quad - F(D) v'(w_0 - P(D) - D) \int_D^{w_0} \big(1 + kg'\big(v(w^{\max}) - v(w_0 - P(D) - D)\big)\big)\, dF(x),\end{aligned} \tag{A11}$$

where $w^{\max} = w_0 - \min(x, E[X])$ (see Equation (A7)). Hence,

$$\begin{aligned}\frac{d^2E[u(w(D))]}{dD^2}\\ &= -f(D) \int_0^D v'(w_0 - P(D) - x)\big(1 + kg'\big(v(w^{\max}) - v(w_0 - P(D) - x)\big)\big)\, dF(x)\\ &\quad + (1 - F(D))^2 \int_0^D v''(w_0 - P(D) - x)\big(1 + kg'\big(v(w^{\max}) - v(w_0 - P(D) - x)\big)\big)\, dF(x)\\ &\quad - (1 - F(D))^2 k \int_0^D v'^2(w_0 - P(D) - x) g''\big(v(w^{\max}) - v(w_0 - P(D) - x)\big)\, dF(x)\\ &\quad - f(D) v'(w_0 - P(D) - D) \int_D^{w_0} \big(1 + kg'\big(v(w^{\max}) - v(w_0 - P(D) - D)\big)\big)\, dF(x)\\ &\quad + F^2(D) v''(w_0 - P(D) - D) \int_D^{w_0} \big(1 + kg'\big(v(w^{\max}) - v(w_0 - P(D) - D)\big)\big)\, dF(x)\\ &\quad - F^2(D) v'^2(w_0 - P(D) - D) k \int_D^{w_0} g''\big(v(w^{\max}) - v(w_0 - P(D) - D)\big)\, dF(x)\\ &\quad + f(D) v'(w_0 - P(D) - D)(1 + kg'(v(w_0 - \min(D, E[X])) - v(w_0 - P(D) - D))).\end{aligned}$$

[5] Schlesinger (1981) emphasizes that expected utility can be a convex function in the deductible level over certain ranges.

At $D = 0$ we get

$$\begin{aligned}
&\left.\frac{d^2 E[u(w(D))]}{dD^2}\right|_{D=0} \\
&= -f(0)v'(w_0 - E[X]) \int_0^{w_0} (1 + kg'(v(w_0 - \min(x, E[X])) - v(w_0 - E[X])))\, dF(x) \\
&\quad + f(0)v'(w_0 - E[X])(1 + kg'(v(w_0) - v(w_0 - E[X]))) \\
&= f(0)v'(w_0 - E[X])k \cdot \begin{pmatrix} g'(v(w_0) - v(w_0 - E[X])) \\ -\int_0^{w_0} g'\big(v\big(w_0 - \min(x, E[X])\big) - v(w_0 - E[X])\big)\, dF(x) \end{pmatrix} \\
&> 0,
\end{aligned}$$

since $g'(v(w_0) - v(w_0 - E[X])) > g'(v(w_0 - \min(x, E[X])) - v(w_0 - E[X]))$ for all $x > 0$. For $\lambda = 0$ we thus have $\frac{dE[u(w(D))]}{dD}|_{D=0} = 0$ and $\frac{d^2 E[u(w(D))]}{dD^2}|_{D=0} > 0$. Convexity at $D = 0$ then implies that $D^* > 0$. Q.E.D.

Proof of Proposition 6: This proposition is true if $\frac{dD_k^*(0)}{dk} > 0$. Applying the same approach shown in Equation (A4), we use the total differential to get

$$\frac{dD_k^*(0)}{dk} = -\frac{\left.\dfrac{\partial^2 E[u(w(D))]}{\partial D \partial k}\right|_{D=D_k^*(0)}}{\left.\dfrac{\partial^2 E[u(w(D))]}{\partial D^2}\right|_{D=D_k^*(0)}}. \tag{A12}$$

In the first step of this proof, we will show that $D_k^*(0)$ is an interior solution (i.e., $0 < D_k^*(0) < w_0$), which implies local concavity (i.e., $\frac{\partial^2 E[u(w(D))]}{\partial D^2}|_{D=D_k^*(0)} < 0$). In the second step, we will show that $\frac{\partial^2 E[u(w(D))]}{\partial D \partial k}|_{D=D_k^*(0)} > 0$, which yields $\frac{dD_k^*(0)}{dk} > 0$. This will complete the proof.

We know from Proposition 5 that $D_k^*(0) > 0$. As $\frac{dE[u(w(D))]}{dD}|_{D=W_0} = 0$ to $\lambda = 0$ (see Equation (A8)) we will show that the RTEU function is convex at $D = w_0$ if $\lambda = 0$ which implies $D_k^*(0) < w_0$. Equation (A9) implies

$$\begin{aligned}
\left.\frac{d^2 E[u(w(D))]}{dD^2}\right|_{D=w_0} &= -f(w_0) \int_0^{w_0} v'(w_0 - x)\big(1 + kg'\big(v(w^{\max}) - v(w_0 - x)\big)\big)\, dF(x) \\
&\quad + f(w_0)v'(0)\big(1 + kg'\big(v(w_0 - E[X]) - v(0)\big)\big).
\end{aligned}$$

Concavity of v implies $v'(0) > v'\ (w_0 - x)$ for all $0 \le x < w_0$. Since $w^{\max} = w_0 - \min(x, E[X])$,

$$g'(v(w_0 - E[X]) - v(0)) > g'\big(v(w^{\max}) - v(w_0 - x)\big) = g'(0)$$

for all $x < E[X]$. For all $x \geq E[X]$, we have

$$g'(v(w_0 - E[X]) - v(0)) > g'\big(v(w^{\max}) - v(w_0 - x)\big) = g'(v(w_0 - E[X]) - v(w_0 - x)).$$

Therefore,

$$\begin{aligned}\frac{d^2 E[u(w(D))]}{dD^2}\bigg|_{D=w_0} &> -f(w_0)v'(0)\big(1 + kg'\big(v(w_0 - E[X]) - v(0)\big)\big)\\ &\quad + f(w_0)v'(0)\big(1 + kg'\big(v(w_0 - E[X]) - v(0)\big)\big)\\ &= 0.\end{aligned}$$

RTEU is thus convex at $D = w_0$ for $\lambda = 0$ and hence $D_k^*(0) < w_0$. Since the maximum is an interior point, RTEU must be locally concave at $D_k^*(0)$. Therefore, Equation (A12) implies that

$$\mathrm{sign}\left(\frac{dD_k^*(0)}{dk}\right) = \mathrm{sign}\left(\frac{\partial^2 E[u(w(D))]}{\partial D \partial k}\bigg|_{D=D_k^*(0)}\right).$$

Differentiating (A8) with respect to k and setting $\lambda = 0$ yields

$$\begin{aligned}\frac{\partial^2 E[u(w(D))]}{\partial D \partial k}\bigg|_{D=D_k^*(0)} &= \big(1 - F\big(D_k^*(0)\big)\big)\int_0^{D_k^*(0)} v'\big(w_0 - P\big(D_k^*(0)\big) - x\big)\\ &\quad \times g'\big(v(w^{\max}) - v\big(w_0 - P\big(D_k^*(0)\big) - x\big)\big)\,dF(x)\\ &\quad - F\big(D_k^*(0)\big)v'\big(w_0 - P\big(D_k^*(0)\big) - D_k^*(0)\big)\\ &\quad \times \int_{D_k^*(0)}^{w_0} g'\big(v(w^{\max}) - v\big(w_0 - P\big(D_k^*(0)\big) - D_k^*(0)\big)\big)\,dF(x).\end{aligned}$$

The FOC $\frac{dE[u(w(D))]}{dD}|_{D=D_k^*(0)} = 0$ implies that for $k > 0$,

$$\begin{aligned}\frac{\partial^2 E[u(w(D))]}{\partial D \partial k}\bigg| D = D_k^*(0) &= -\frac{1}{k}\cdot\left[\begin{matrix}\big(1-F\big(D_k^*(0)\big)\big)\int_0^{D_k^*(0)} v'\big(w_0 - P\big(D_k^*(0)\big) - x\big)\,dF(x)\\ -F\big(D_k^*(0)\big)v'\big(w_0 - P\big(D_k^*(0)\big) - D_k^*(0)\big)\big(1 - F\big(D_k^*(0)\big)\big)\end{matrix}\right]\\ &> -\frac{1}{k}v'\big(w_0 - P\big(D_k^*(0)\big) - D_k^*(0)\big)\\ &\quad \cdot\big[\big(1 - F\big(D_k^*(0)\big)\big)F\big(D_k^*(0)\big) - F\big(D_k^*(0)\big)\big(1 - F\big(D_k^*(0)\big)\big)\big]\\ &= 0.\end{aligned}$$

Hence $\mathrm{sign}(\frac{\partial^2 E[u(w(D))]}{\partial D \partial k}|_{D=D_k^*(0)}) > 0$, and therefore, $\frac{dD_k^*(0)}{dk} > 0$. Q.E.D.

Proof of Proposition 7: This proof is in two parts. First, we will show that for $k = 0$, the individual purchases no insurance at $\lambda = \bar{\lambda}$. Then, we will show that at that same $\lambda = \bar{\lambda}$, the individual purchases some insurance when $k > 0$.

We begin by recalling from Equation (5) that because $F(w_0) = 1$, $P'(w_0) = 0$. Therefore, from Equation (A8), $\frac{dE[u(w(D))]}{dD}|_{D=w_0} = 0$ for all $\lambda, k \geq 0$. We also know from Equation (A10) that $\frac{dE[u(w(D))]}{dD}|_{D=0} > 0$ for all $\lambda > 0$, including $\lambda = \bar{\lambda}$. To show that $D_0^*(\bar{\lambda}) = w_0$, it is sufficient to prove that $\frac{dE[u(w(D))]}{dD} > 0$ for all $0 < D < w_0$. For $k = 0$ and $\lambda = \bar{\lambda}$, the first derivative (A8) is

$$\begin{aligned}
\frac{dE[u(w(D))]}{dD} \\
&= (1+\bar{\lambda})(1-F(D))\int_0^D v'(w_0 - P(D) - x)\,dF(x) \\
&\quad - (1-(1+\bar{\lambda})(1-F(D)))v'(w_0 - P(D) - D)(1-F(D)) \\
&= (1+\bar{\lambda})(1-F(D))\left[\int_0^D v'(w_0 - P(D) - x)\,dF(x) + v'(w_0 - P(D) - D)(1-F(D))\right] \\
&\quad - v'(w_0 - P(D) - D)(1-F(D)) \\
&= (1-F(D))v'(0)\cdot \frac{\displaystyle\int_0^D v'(w_0 - P(D) - x)\,dF(x) + v'(w_0 - P(D) - D)(1-F(D))}{\displaystyle\int_0^{w_0} v'(w_0 - x)\,dF(x)} \\
&\quad - v'(w_0 - P(D) - D)(1-F(D)) \\
&= v'(w_0 - P(D) - D)(1-F(D)) \\
&\quad \times \left[\frac{\dfrac{v'(0)}{v'(w_0 - P(D) - D)}\left(\displaystyle\int_0^D v'(w_0 - P(D) - x)\,dF(x)\right) + v'(0)(1-F(D))}{\displaystyle\int_0^{w_0} v'(w_0 - x)\,dF(x)} - 1\right] \\
&> 0.
\end{aligned} \tag{A13}$$

This inequality holds because $\frac{v'(0)}{v'(w_0 - P(D) - D)} > 1$, $v'(w_0 - P(D) - x) > v'(w_0 - x)$ for all x, and $v'(0)(1 - F(D)) > \int_D^{w_0} v'(w_0 - x)\,dF(x)$.

We have thus shown that at $\lambda = \bar{\lambda}$, RTEU is strictly increasing in D when $k = 0$. Therefore, $D_0^*(\bar{\lambda}) = w_0$ and it is optimal for the individual not to buy any insurance.

To show that a regret-averse individual ($k > 0$) buys some insurance at this loading factor ($D_k^*(\bar{\lambda}) < w_0$) we will prove that RTEU is convex at $D = w_0$. From Equation (A9), the second derivative at $D = w_0$ for $k > 0$ and $\lambda = \bar{\lambda}$ is

$$
\begin{aligned}
&\left.\frac{d^2E[u(w(D))]}{dD^2}\right|_{D=w_0} \\
&= -(1+\bar{\lambda})f(w_0)\int_0^{w_0} v'(w_0-x)\big(1+kg'\big(v(w^{\max})-v(w_0-x)\big)\big)\,dF(x) \\
&\quad + f(w_0)v'(0)\big(1+kg'\big(v\big(w^{\max}(w_0)\big)-v(0)\big)\big) \\
&= -(1+\bar{\lambda})f(w_0)k\int_0^{w_0} v'(w_0-x)g'\big(v(w^{\max})-v(w_0-x)\big)\,dF(x) \\
&\quad + f(w_0)v'(0)kg'\big(v\big(w^{\max}(w_0)\big)-v(0)\big) \\
&= -f(w_0)kv'(0)\cdot\frac{\displaystyle\int_0^{w_0} v'(w_0-x)g'\big(v(w^{\max})-v(w_0-x)\big)\,dF(x)}{\displaystyle\int_0^{w_0} v'(w_0-x)\,dF(x)} \\
&\quad + f(w_0)v'(0)kg'\big(v\big(w^{\max}(w_0)\big)-v(0)\big) \\
&= f(w_0)kv'(0)g'\big(v\big(w^{\max}(w_0)\big)-v(0)\big) \\
&\quad \cdot\left[1-\frac{\displaystyle\int_0^{w_0} v'(w_0-x)g'\big(v(w^{\max})-v(w_0-x)\big)\,dF(x)}{g'\big(v\big(w^{\max}(w_0)\big)-v(0)\big)\displaystyle\int_0^{w_0} v'(w_0-x)\,dF(x)}\right],
\end{aligned}
$$

where

$$w^{\max}(w_0) = w_0 - [(1+\bar{\lambda})E[(X-\bar{D}(\bar{\lambda}))^+] + \bar{D}(\bar{\lambda})].$$

Hence, for all $x < (1+\bar{\lambda})E[(X-\bar{D}(\bar{\lambda}))^+] + \bar{D}(\bar{\lambda})$ we have

$$g'\big(v\big(w^{\max}(w_0)\big)-v(0)\big) > g'\big(v(w^{\max})-v(w_0-x)\big) = g'(0),$$

and for all $x \geq (1+\bar{\lambda})E[(X-\bar{D}(\bar{\lambda}))^+] + \bar{D}(\bar{\lambda})$,

$$g'\big(v\big(w^{\max}(w_0)\big)-v(0)\big) > g'\big(v(w^{\max})-v(w_0-x)\big) = g'\big(v\big(w^{\max}(w_0)\big)-v(w_0-x)\big).$$

Therefore,

$$\frac{\displaystyle\int_0^{w_0} v'(w_0-x)g'\big(v(w^{\max})-v(w_0-x)\big)\,dF(x)}{g'\big(v\big(w^{\max}(w_0)\big)-v(0)\big)\displaystyle\int_0^{w_0} v'(w_0-x)\,dF(x)} < 1,$$

and so $\frac{d^2E[u(w(D))]}{dD^2}|_{D=w_0} > 0$. Convexity at $D = w_0$ and the fact that $\frac{dE[u(w(D))]}{dD}|_{D=w_0} = 0$ implies that the optimal deductible level is an interior point $D_k^*(\bar{\lambda}) < w_0$ for all $k > 0$.

Analogously to (A12), we deduce

$$\frac{dD_k^*(\bar{\lambda})}{dk} = -\frac{\left.\frac{\partial^2 E[u(w(D))]}{\partial D \partial k}\right|_{D=D_k^*(\bar{\lambda})}}{\left.\frac{\partial^2 E[u(w(D))]}{\partial D^2}\right|_{D=D_k^*(\bar{\lambda})}}.$$

RTEU is locally concave in the deducible level at $D = D_k^*(\bar{\lambda})$ as $D_k^*(\bar{\lambda})$ is an interior solution.

$\frac{\partial^2 E[u(w(D))]}{\partial D^2}|_{D=D_k^*(\bar{\lambda})} < 0$ therefore implies

$$\text{sign}\left(\frac{dD_k^*(\bar{\lambda})}{dk}\right) = \text{sign}\left(\left.\frac{\partial^2 E[u(w(D))]}{\partial D \partial k}\right|_{D=D_k^*(\bar{\lambda})}\right).$$

Differentiation of (A8) with respect to k and setting $\lambda = \bar{\lambda}$ yields

$$\begin{aligned}\left.\frac{\partial^2 E[u(w(D))]}{\partial D \partial k}\right|_{D=D_k^*(\bar{\lambda})} &= (1+\bar{\lambda})\big(1-F\big(D_k^*(\bar{\lambda})\big)\big)\int_0^{D_k^*(\bar{\lambda})} v'\big(w_0 - P\big(D_k^*(\bar{\lambda})\big) - x\big) \\ &\quad \times g'\big(v(w^{\max}) - v\big(w_0 - P\big(D_k^*(\bar{\lambda})\big) - x\big)\big)\, dF(x) \\ &\quad - (1+\bar{\lambda})F\big(D_k^*(\bar{\lambda})\big)v'\big(w_0 - P\big(D_k^*(\bar{\lambda})\big) - D_k^*(\bar{\lambda})\big) \\ &\quad \times \int_{D_k^*(\bar{\lambda})}^{w_0} g'\big(v(w^{\max}) - v\big(w_0 - P\big(D_k^*(\bar{\lambda})\big) - D_k^*(\bar{\lambda})\big)\big)\, dF(x).\end{aligned}$$

The FOC $\frac{\partial E[u(w(D))]}{\partial D}|_{D=D_k^*(\bar{\lambda})} = 0$ implies for $k > 0$,

$$\begin{aligned}&\left.\frac{\partial^2 E[u(w(D))]}{\partial D \partial k}\right|_{D=D_k^*(\bar{\lambda})} \\ &= -\frac{1}{k}(1+\bar{\lambda})\left(1-F\left(D_k^*(\bar{\lambda})\right)\right)\cdot\left[\begin{array}{l}\int_0^{D_k^*(\bar{\lambda})} v'\big(w_0 - P\big(D_k^*(\bar{\lambda})\big) - x\big)\, dF(x) \\ + v'\big(w_0 - P\big(D_k^*(\bar{\lambda})\big) - D_k^*(\bar{\lambda})\big)\left(1-F\left(D_k^*(\bar{\lambda})\right)\right)\end{array}\right] \\ &\quad + \frac{1}{k}v'\big(w_0 - P\big(D_k^*(\bar{\lambda})\big) - D_k^*(\bar{\lambda})\big)\left(1-F\left(D_k^*(\bar{\lambda})\right)\right)\end{aligned}$$

$$
\begin{aligned}
&= -\frac{1}{k}\left(1 - F\left(D_k^*(\bar{\lambda})\right)\right) v'(0) \\
&\quad \times \frac{\int_0^{D_k^*(\bar{\lambda})} v'\big(w_0 - P\big(D_k^*(\bar{\lambda})\big) - x\big)\, dF(x) + v'\big(w_0 - P\big(D_k^*(\bar{\lambda})\big) - D_k^*(\bar{\lambda})\big)\left(1 - F\big(D_k^*(\bar{\lambda})\big)\right)}{\int_0^{w_0} v'(w_0 - x)\, dF(x)} \\
&\quad + \frac{1}{k} v'\big(w_0 - P\big(D_k^*(\bar{\lambda})\big) - D_k^*(\bar{\lambda})\big)\left(1 - F\big(D_k^*(\bar{\lambda})\big)\right) \\
&= -\frac{1}{k} v'\big(w_0 - P\big(D_k^*(\bar{\lambda})\big) - D_k^*(\bar{\lambda})\big)\left(1 - F\big(D_k^*(\bar{\lambda})\big)\right) \\
&\quad \times \left[\frac{\frac{v'(0)}{v'\big(w_0 - P\big(D_k^*(\bar{\lambda})\big) - D_k^*(\bar{\lambda})\big)}\left(\int_0^{D_k^*(\bar{\lambda})} v'\big(w_0 - P\big(D_k^*(\bar{\lambda})\big) - x\big)\, dF(x)\right) + v'(0)\left(1 - F\big(D_k^*(\bar{\lambda})\big)\right)}{\int_0^{w_0} v'(w_0 - x)\, dF(x)} - 1\right] \\
&< 0,
\end{aligned}
$$

where the last inequality follows from (A13). Therefore sign$\left(\frac{\partial^2 E[u(w(D))]}{\partial D \partial k}\big|_{D=D_k^*(\bar{\lambda})}\right) < 0$ and hence $\frac{dD_k^*(\bar{\lambda})}{dk} < 0$.

Proof of Proposition 8: For an arbitrarily fixed $k > 0$, define $\Delta_k(\lambda) = D_0^*(\lambda) - D_k^*(\lambda)$. We have $\Delta_k(0) < 0$ from Proposition 5 and $\Delta_k(\bar{\lambda}) > 0$ from Proposition 7. Since $\Delta_k(\lambda)$ is a continuous function in λ, the Intermediate Value Theorem implies that there must exist a $\hat{\lambda}_k$ with $0 < \hat{\lambda}_k < \bar{\lambda}$ such that

$$\Delta_k(\hat{\lambda}_k) = 0,$$

so

$$D_k^*(\hat{\lambda}_k) = D_0^*(\hat{\lambda}_k).$$

We next show that this intermediate loading factor $\hat{\lambda}_k$ is independent of k. As $D_k^*(\hat{\lambda}_k)$ is an inner solution, $\hat{\lambda}_k$ is determined by the following FOC (see Equation (A8)) for $k = 0$

$$
\begin{aligned}
&-P'\big(D_0^*(\hat{\lambda}_k)\big) \int_0^{D_0^*(\hat{\lambda}_k)} v'\big(w_0 - P\big(D_0^*(\hat{\lambda}_k)\big) - x\big)\, dF(x) \\
&- \big(P'\big(D_0^*(\hat{\lambda}_k)\big) + 1\big) v'\big(w_0 - P\big(D_0^*(\hat{\lambda}_k)\big) - D_0^*(\hat{\lambda}_k)\big)\big(1 - F\left(D_0^*(\hat{\lambda}_k)\right)\big) \\
&= 0,
\end{aligned}
$$

and for $k > 0$

$$
\begin{aligned}
&-P'\big(D_k^*(\hat{\lambda}_k)\big)\int_0^{D_k^*(\hat{\lambda}_k)} v'\big(w_0 - P\big(D_k^*(\hat{\lambda}_k)\big) - x\big) \\
&\quad \times \big(1 + kg'\big(v(w^{\max}) - v\big(w_0 - P\big(D_k^*(\hat{\lambda}_k)\big) - x\big)\big)\big)\, dF(x) \\
&- \big(P'\big(D_k^*(\hat{\lambda}_k)\big) + 1\big)v'\big(w_0 - P\big(D_k^*(\hat{\lambda}_k)\big) - D_k^*(\hat{\lambda}_k)\big) \\
&\times \int_{D_k^*(\hat{\lambda}_k)}^{w_0} \big(1 + kg'\big(v(w^{\max}) - v\big(w_0 - P\big(D_k^*(\hat{\lambda}_k)\big) - D_k^*(\hat{\lambda}_k)\big)\big)\big)\, dF(x) \\
= 0.
\end{aligned}
$$

As $D_k^*(\hat{\lambda}_k) = D_0^*(\hat{\lambda}_k)$ those two conditions reduce to the following:

$$
\begin{aligned}
&-P'\big(D_0^*(\hat{\lambda}_k)\big)\int_0^{D_0^*(\hat{\lambda}_k)} v'\big(w_0 - P\big(D_0^*(\hat{\lambda}_k)\big) - x\big) \\
&\quad \times g'\big(v(w^{\max}) - v\big(w_0 - P\big(D_0^*(\hat{\lambda}_k)\big) - x\big)\big)\, dF(x) \\
&- \big(P'\big(D_0^*(\hat{\lambda}_k)\big) + 1\big)v'\big(w_0 - P\big(D_0^*(\hat{\lambda}_k)\big) - D_0^*(\hat{\lambda}_k)\big) \\
&\times \int_{D_0^*(\hat{\lambda}_k)}^{w_0} g'\big(v(w^{\max}) - v\big(w_0 - P\big(D_0^*(\hat{\lambda}_k)\big) - D_0^*(\hat{\lambda}_k)\big)\big)\, dF(x) \\
= 0.
\end{aligned}
$$

As this condition is independent of k, $\hat{\lambda}_k = \hat{\lambda}$ for all $k > 0$ and thus $D_k^*(\hat{\lambda}) = D_0^*(\hat{\lambda})$ for all $k \geq 0$. Q.E.D.

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