Transaction Attributes and Customer Valuation

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Abstract

Dynamic customer targeting is a common task for marketers actively managing customer relationships. Such efforts can be guided by insight into the return on investment from marketing interventions, which can be derived as the increase in the present value of a customer's expected future transactions. Using the popular latent attrition framework, one could estimate this value by manipulating the levels of a set of nonstationary covariates. We propose such a model that incorporates transaction-specific attributes and maintains standard assumptions of unobserved heterogeneity. We demonstrate how firms can approximate an upper bound on the appropriate amount to invest in retaining a customer and demonstrate that this amount depends on customers' past purchase activity, namely the recency and frequency of past customer purchases. Using data from a B2B service provider as our empirical application, we apply our model to estimate the revenue lost by the service provider when it fails to deliver a customer's requested level of service. We also show that the lost revenue is larger than the corresponding expected gain from exceeding a customer's requested level of service. We discuss the implications of our findings for marketers in terms of managing customer relationships.

Keywords: services marketing, customer retention, probability models, marketing ROI, customer value

According to IBM's recent survey of chief marketing officers, 63% of respondents said that return on investment (ROI) would be the most important measure of success in the next three to five years (IBM Corporation 2011). Braun and Schweidel (2011) argue that marketing ROI should be measured in terms of an expected change in the residual value of the customer base that occurs from a marketing intervention. Latent attrition models, such as the the Pareto/NBD (Schmittlein, Morrison, and Colombo 1987) and the BG/NBD (Fader, Hardie, and Lee 2005a), are useful for estimating residual value, but they do not lend themselves easily to incorporating covariates that change from transaction to transaction. In the marketing ROI domain, possible examples of these covariates include attributes of the marketing mix, exogenous conditions at the time of the transaction (e.g., weather effects), or an investment in improved customer experiences.

The quality of customer information to which a firm has access, which encompasses being both broad and up-to-date, has been found to moderate the profitability of customer prioritization (Homburg, Droll, and Totzek 2008). Similarly, Mithas, Krishnan, and Fornell (2005) posit that the value of customer relationship management tools lies in their ability to facilitate learning about customers over the course of multiple interactions, the insights from which can then be used to target customers dynamically with tailored offerings. Though some researchers have proposed methods of incorporating both time-invariant and time-variant predictors into customer base analyses (Abe 2009; Schweidel and Knox 2013), these models stop short of establishing the link between the effects of transaction-specific attributes and forecasts of residual value. Thus, how to use information about transaction attributes to assess the increased customer value stemming from marketing efforts remains an open and important area of research.

In this paper, we propose a latent attrition model that integrates transaction attributes into a probability model of customer retention and lifetime value. One application of this model (and the one we use in our empirical analysis) is to evaluate the impact of a customer's service experience on customer retention. Consider the following scenario. A local business (i.e., the customer) contracts with a service provider to meet a recurring business need, such as copywriting services. Based on examples of the provider's work, the customer has some expectation of the caliber and timeliness of the work. If the provider delivers a top-notch experience, the customer may engage the provider the next time he needs similar services. By increasing the retention probability for that customer, all else being equal, the service experience delivered by the provider's team may have generated additional revenue in subsequent time periods. In contrast, if the provider falls short and delivers a sub-par experience, the customer may be more likely to search for another provider when the need for similar services arises in the future. When this happens, the future revenue stream from the customer falls to zero. In this example, the attributes of a single customer-firm interaction could affect the future revenue stream. The "transaction attribute" in this case is an indicator of the customer's service experience. More generally, transaction attributes may refer to some aspect of the marketing mix, the employee who processed a customer's transaction, or any information about a customer's transaction that the firm has collected.

While there is an intuitive relationship between transaction attributes and retention probabilities, and hence customer value, extant empirical research often fails to differentiate among transactions other than with regard to the times at which the events occur. Models like the Pareto/NBD and the BG/NBD rely on summary statistics of past transactional activity, namely the total number of transactions (frequency) and the time of the most recent transaction (recency). However, by aggregating the data to this level, information about other characteristics of each transaction is lost. Take the case of two customers with the same number of transactions during the last year, and with their most recent transaction occurring on the same day. Ignoring attributes of those transactions, these two customers' transaction histories, and the resulting predictions of residual value, are identical. If a firm had access to additional information about the last customer-firm touch point, the firm could have different beliefs of what each customer would do in the future. Failing to meet stated and established standards of service, for example, may lead the firm to believe that a customer is now at a greater risk for churn, while exceeding these standards may lead to perceptions that this risk is lower (Ho, Park, and Zhou 2006).

Exploiting information about the attributes of each transaction gives firms additional guidance for managing customer relationships, compared to the information provided by frequency and recency alone. The value of transaction attribute data comes from how that information affects the firm's decisions. We assess the effect of a particular transaction attribute in terms of the projected change in discounted expected residual transactions (DERT, Fader, Hardie, and Lee 2005b; Fader, Hardie, and Shang 2010). DERT is the present value of expected future transactions, taking into account the probability that a customer may have already churned, or will churn in the future. Differences in the transaction attributes will produce variation in DERT beyond that which is captured by the recency and frequency of transactions. Attribute data can affect firm decisions in two ways. First, ignoring attributes can lead to different estimates of DERT, which in turn may lead to a suboptimal decisions based on bad information. Second, the "incremental DERT" that comes from manipulating the transaction attributes (say, by increasing investment in retaining a specific customer immediately before a transaction) can serve as an upper limit for such a marketing investment (Braun and Schweidel 2011). For example, if a firm could estimate the effect of a marketing mix variable on churn, the incremental DERT would be the difference between the forecast of DERT under the marketing treatment, and a baseline DERT without it. To the best of our knowledge, our research is the first to examine the value of information about transaction attributes in terms of the change in expected future customer transactions.

In the next section, we provide a brief discussion of relevant literature in the customer base analysis and service quality areas, the latter of which relates to the context of our empirical example. We show how our research adds to the associated body of knowledge. Then, we describe the model, in terms of a likelihood function, as well as derive both prior and posterior DERT that can be expressed in either closed-form or as a summation. The subsequent empirical analysis, which employs a dataset from a noncontractual service provider, illustrates a case in which including transaction attribute data adds predictive power to the model. Finally, we show that incremental DERT has a nonlinear and non-monotonic relationship with customer recency and frequency. The patterns suggest that falling short of the requested service level has a smaller effect on retaining customers who are either likely to have already churned, or who are highly *unlikely* to have churned, compared to the effect on customers for whom there is more uncertainty in their active status.

1 Related Literature

Latent attrition models such as the Pareto/NBD and the BG/NBD are workhorse models of customer base analysis (Schmittlein, Morrison, and Colombo 1987; Fader, Hardie, and Lee 2005a; Fader, Hardie, and Lee 2005b). A notable limitation of this class of models is the difficulty of incorporating information about attributes that accompany each customer-firm interaction, such as marketing actions that vary across transactions. Recognizing this gap in the literature, Ho, Park, and Zhou (2006) propose incorporating customer satisfaction into the latent attrition framework. Their model assumes that customer satisfaction affects the rate at which customers conduct transactions, and they demonstrate how satisfaction can be allowed to impact the attrition process. Although Ho, Park, and Zhou do consider information that is specific to the customer-firm interaction (i.e., satisfaction), their model is analytic, as opposed to empirical. Nevertheless, they illustrate the importance of incorporating customer satisfaction and, more broadly, customer-firm interaction information, into estimates of customer value.

Another notable difference between the Ho, Park, and Zhou (2006) model and empirical latent attrition models is that Ho, Park, and Zhou assume homogeneous purchase and attrition processes. In addition to capturing variation across customers, models that allow for unobserved heterogeneity let firms update their expectations of customer behavior as new data become available. These posterior inferences are necessary both for valuing customers (Fader, Hardie, and Lee 2005a; Braun and Schweidel 2011) and for assessing the impact of marketing efforts. Schweidel and Knox (2013) illustrate this idea with a joint model of individuals' donation activity and the direct marketing efforts of a non-profit organization, accounting for the potentially non-random nature of marketing efforts. In their example, the authors allow for direct marketing activity to affect the likelihood of donation each month, the amount of a donation conditional on the donation occurring, and the likelihood with which a donor becomes inactive. To account for unobserved heterogeneity, Schweidel and Knox apply a latent class structure. While their model allows for marketing actions to impact each of the processes that may affect customer value, they do not consider how an individual's transaction history may affect expectations of future activity. Moreover, they do not consider how their framework could be adapted to estimate measures such as expectations of future purchases, customer lifetime value, or residual value.

Like Schweidel and Knox (2013), Knox and van Oest (2014) also employ a latent class structure to account for heterogeneity across customers in their investigation of the impact of customer complaints on customer churn. They assess the impact of customer complaints and recovery by the firm for two types of customers: a new customer and an established customer. The authors demonstrate that the residual value of customers following a complaint varies with both the customers' past purchase activity and past complaints. The authors distinguish between the effects of marketing interventions on new and established customers, consistent with research that has investigated the profitability of behavior-based marketing actions (Villas-Boas 1999; Pazgal and Soberman 2008; Shin and Sudhir 2010). While extant work has documented the benefits of differentiating between new and established customers, such work often does not seek to provide insight into how marketing interventions may affect established customers with different transaction histories. We contribute to this stream of research by developing a modeling framework that allows us to conduct a systematic investigation into how customers' recency and frequency of past transactions (Fader, Hardie, and Lee 2005b) affects the incremental impact of marketing efforts, which can enable marketers to target customers with increased precision.

Although there are many different attributes a transaction could possess, our empirical analysis in Section 3 is in the domain of service quality. Several researchers have studied service quality and its relationship to customer expectations. Boulding et al. (1993) find that a customer's evaluation of a service encounter is affected by his prior expectations of what *will* and *should* occur, as well as the quality of service delivered on recent service encounters. In essence, *will* and *should* expectations for a service encounter are a weighted average of prior expectations and the recently experienced service. Boulding, Kalra, and Staelin (1999) further investigate the process by which expectations are updated. In addition to affecting a customer's cumulative opinion, the authors find that prior beliefs also affect how experiences are viewed. As a result, prior expectations deliver a "double whammy" to evaluations of quality. This suggests that service encounters are not all equal in the eyes of consumers, as the way in which service encounters are viewed are affected by past experiences. For example, the exact same level of quality might exceed expectations in a mid-range family restaurant, but miss expectations in a fancy bistro. Yet, extant customer valuation models in both non-contractual and contractual settings often assume that the “touch points” associated with customer-firm interactions are equivalent to each other.

Rust et al. (1999) further investigate the role of customer expectations in perceptions of quality. Rather than focusing on the average expectation across customers, the authors highlight the importance of the distribution of customer expectations. They tackle a number of myths that had been held with regard to the level of service that providers should deliver to their customers. In contrast to the popularly held belief that firms must exceed expectations, the authors find evidence that simply meeting customers’ expectations can result in a positive shift in preferences. They also find that service encounters that are slightly below expectations may not affect customers’ preferences at all. Though provoking, the authors recognize that because they conducted their investigation in a laboratory setting, and relied on self reports, there is a need for additional research.

In addition to the work that has been conducted on service quality, our research is also related to work on customer satisfaction. Bolton (1998) investigates the impact of customer satisfaction on the duration for which customers continue to subscribe to a contractual service. She finds that reported customer satisfaction with the service, solicited prior to the decision of whether to remain a subscriber or cancel service, is positively related to the duration for which a customer will retain service. She also finds evidence that recent experiences with the service provider are weighed differently depending on whether the experience was evaluated as positive or negative. To the best of our knowledge, research on customer valuation has not incorporated this differential weighting of customer experiences into estimates of customers’ future behavior.

2 Model

In this section we propose a general form of a latent attrition model that incorporates transaction attributes. To keep terminology consistent with the empirical example in Section 3, we say that the customer of the firm places orders for jobs, and the firm fills those orders by completing the jobs. Thus, orders and jobs always occur in a pair, and are indexed by k. We assume that these jobs are completed the instant the order is placed, so we index calendar time for orders and jobs by t. Without loss of generality, we define a unit of calendar time as one week. The service was introduced to the marketplace at time $t = 0$ and T is the week of the end of the observation period. Let t_1 be the week of the customer's first order, let x be the number of orders between times t_1 and T, *including* that first order at t_1, and let t_k be the time of order k. Therefore, t_x is order time of the final, observed job. For clarity, we are suppressing the customer-specific indices on t and x in the model exposition.

Our baseline model is a variant of the BG/NBD model for non-contractual customer base analysis (Fader, Hardie, and Lee 2005a). Immediately before the customer places an initial order at time t_1, he is in an active state. While active, the customer places orders according to a Poisson process with rate λ. After each job (including the first one), a customer may churn, resulting in that order being his last. With probability p_k, the customer churns after the k^{th} job and transitions from the active state to the inactive state. Upon doing so, we assume that the customer is lost for good and will not place any more orders, ever. If the customer does not churn, then the time until the next order, $t_{k+1} - t_k$, is a realization of an exponential random variable with rate λ. We never observe directly when, or if, a customer churns, although if a customer places x orders, he must have survived $x - 1$ possible churn opportunities.

For a customer who places x orders between times t_1 and T, the joint density of the $x - 1$ inter-order times is the product of $x - 1$ exponential densities. For this customer, there could not have been any orders between times t_x and T. This "hiatus" could occur in one of two ways. One possibility is that the customer may have churned after job x, with probability p_x. Alternatively, the customer may have "survived" with probability $1 - p_x$, but the time of the next order would be sometime after T. Thus, conditional on surviving x jobs, the probability of not observing any more jobs before time T is $e^{-\lambda(T-t_x)}$. Hence, the conditional data likelihood for a single customer is

$$f(x, t_{2:x}|\lambda, p_{1:x}) = \lambda^{x-1} e^{-\lambda(t_x - t_1)} \left[\prod_{k=1}^{x-1} (1-p_k)\right] \left[p_x + (1-p_x)\, e^{-\lambda(T-t_x)}\right] \tag{1}$$

If p_k were time-invariant (i.e., the same for all k), Equation 1 would be the individual-level likelihood in the BG/NBD[1]. To incorporate transaction-specific information, we allow p_k to vary across orders in our model. We define the probability of becoming inactive by transitioning to the inactive state immediately after job k as $p_k = 1 - e^{-\theta q_k}$ and define $B_k = \sum_{j=1}^{k} q_k$, where q_k is a non-negative scalar value that can influence the probability that a customer transitions to the inactive state after job k. If we restrict $q_k = 1$ for all k, then $p = 1 - e^{-\theta}$, or alternatively, $\theta = -\log(1-p)$. The expression q_k, and hence B_k, could be a function of further parameters and observed data, such as the transaction attributes. For example, we might give q_k a log-linear structure, where $\log q_k = \beta' z_k$, β is a vector of homogeneous coefficients, and z_k is a vector of covariates that represents attributes of transaction k. Substituting these definitions into Equation 1,

$$f(x, t_{2:x}|\lambda, \theta, q_{1:x}) = \lambda^{x-1} e^{-\lambda(t_x - t_1) - \theta B_{x-1}} \left[1 - e^{-\theta q_x}\left(1 - e^{-\lambda(T-t_x)}\right)\right] \tag{2}$$

The expression of the likelihood in Equation 2 assumes that all customers place orders at the same rate, and that all customers have the same baseline propensity to churn after each job. To incorporate heterogeneity of latent characteristics into the model, we let λ and θ vary across the population according to gamma distributions, where $\lambda \sim \mathcal{G}_\lambda(r, a)$ and $\theta \sim \mathcal{G}_\theta(s, b)$. Integrating over these latent parameters, we get the marginal likelihood:

$$\mathcal{L} = \frac{\Gamma(r+x-1)}{\Gamma(r)} \frac{a^r}{(a+t_x-t_1)^{r+x-1}} \left(\frac{b}{b+B_{x-1}}\right)^s \left[1 - \left(\frac{b+B_{x-1}}{b+B_x}\right)^s \left(1 - \left(\frac{a+t_x-t_1}{a+T-t_1}\right)^{r+x-1}\right)\right] \tag{3}$$

A detailed derivation of the marginal likelihood is in Equation 10 in Appendix A. Transforming

[1]As with the BG/NBD, in our model a high transaction rate suggests additional attrition opportunities.

a gamma-distributed random variable to yield a value between zero and one was discussed by Grassia (1977). If p_k were constant across time, and varied across the population according to a beta distribution, then the marginal likelihood would be the same as the BG/NBD. Griffiths and Schafer (1981) show that Grassia's method and a beta distribution are "practically identical," and that choice between them could be based "entirely on mathematical convenience." Our approach lets us estimate model parameters using standard maximum likelihood techniques even when the attrition probability depends on transaction-specific covariates.

Certain design decisions allow us to maintain some degree of computational efficiency. We allow for unobserved heterogeneity in λ and θ by carefully choosing a parametric family of independent mixing distributions. The B_x term is heterogeneous across observable characteristics that vary across both individuals and time. Also, we allow for unobserved nonstationarity in some of the parameters in B_x (see Equations 8 and 9).[2]

2.1 Conditional expectations and DERT

Once a manager has parameter estimates in hand, he might be interested in the number of orders that we might receive from a newly acquired customer during a period of t weeks. In Appendix A we show that the *prior* expected value of this order count is

$$E[X(t)] = \sum_{k=1}^{\infty} \left(\frac{b}{b + B_k}\right)^s \widetilde{\mathbb{B}} \left(\frac{t}{a + t}; k, r\right) \tag{4}$$

Therefore, the manager can estimate the expected number of orders by truncating this infinite series. The function $\widetilde{\mathbb{B}}(x; a, b)$ is regularized beta function, which also happens to be the cdf of a beta distribution, with parameters a and b, evaluated at x.[3].

One way to interpret the summation in Equation 4 is as the sum of the probabilities of ordering

[2]Among the alternative specifications considered was one in which we allow for heterogeneity in β via a latent class structure. The estimated probability of being in the first latent class was $p = 1$, suggesting that the additional model complexity is not warranted. We also estimated a model that allows λ to vary with z_k. The Hessian in the resulting model was singular, so the parameters could not be identified when covariates are assumed to impact both the transaction rate and the attrition process simultaneously.

[3]A glossary of many of the functions we use in this paper is in Table 3 in Appendix A

k jobs before time t, for all possible values of k. These jobs are hypothetical, so we need a model for each q_k that comprises B_k, which is the cumulative sum of $q_1, q_2, \ldots, q_k$. In general, one could simulate multiple sequences of q_k from that model, truncated at a sufficiently large value of k, and then average $E[X(t)]$ across sequences. An alternative heuristic is to replace each B_k with its mean. This approximation will be most accurate when the variance in B_k is very small, which, as we will show later, is the case in our empirical application. This approximation is not needed to estimate the model itself, but only to calculate the expected number of transaction without resorting to the use of simulations.

A conceptually useful expression is the probability that a customer is still active at time T.

$$P(\mathcal{A}) = \left[1 - \left(\frac{a + T - t_1}{a + t_x - t_1}\right)^{r+x-1} \left(1 - \left(\frac{b + B_x}{b + B_{x-1}}\right)^{s}\right)\right]^{-1} \tag{5}$$

The derivation for Equation 5 is in Appendix A.

A manager might also want to know how many orders he can expect from an existing customer, during the next t^* periods, given an observed transaction history. In Appendix A, we show that this *posterior* expected number of future transactions is

$$E^*[X(t^*)|x, t_x] = P(\mathcal{A}) \times \sum_{k=1}^{\infty} \left(\frac{B_x + b}{B_x + B_k + b}\right)^{s} \widetilde{\mathbb{B}}\left(\frac{t^*}{t^* + a + T - t_1}; k, r + x - 1\right) \tag{6}$$

In Equation 6, the index of the summation k refers to the potential orders that are made after time T. As discussed previously, we can either model B_k so that we may simulate future values of B_k explicitly, or substitute $E(B_k)$ as an approximation. The prior and posterior probability mass functions for the number of orders (i.e., to express the probability of placing a particular number of orders during some future number of weeks) are included in Appendix B, which is available as part of the online supplement.

While it is useful to know the expected number of future orders, orders are placed at different times. One order may be placed at time $T + 1$, the next order may not be placed until a point in time that is well into the future. Given the time value of money, orders that occur soon are more valuable than orders that are placed later. Therefore, an appropriate metric for the expected number of a customer's future transactions should discount those transactions back to the present. The value for discounted expected residual transactions (DERT) is proportional to a customer's residual lifetime value when the margin is constant (Fader, Hardie, and Shang 2010). Let δ be a discount factor that captures the time value of money, so a dollar earned t weeks from now is worth δ^t today (for notational simplicity, we reset the counter of t so $t = 0$ at T, and we assume that payments are made at the end of the week). The posterior estimate for the DERT of this customer is the sum of discounted incremental expected orders.

$$
\begin{aligned}
DERT &= \sum_{t=1}^{\infty} \Big(E^*[X(t)|x, t_x, B_x, T] - E^*[X(t-1)|x, t_x, B_x, T] \Big) \delta^t \\
&= P(\mathcal{A}) \sum_{k=1}^{\infty} \left(\frac{B_x + b}{B_x + B_k + b} \right)^s \\
&\quad \times \sum_{t=1}^{\infty} \delta^t \left[\widetilde{\mathbb{B}} \left(\frac{t}{a + T - t_1 + t}; k, r + x - 1 \right) - \widetilde{\mathbb{B}} \left(\frac{t-1}{a + T - t_1 + t - 1}; k, r + x - 1 \right) \right] \\
&= (1 - \delta) P(\mathcal{A}) \sum_{k=1}^{\infty} \left(\frac{B_x + b}{B_x + B_k + b} \right)^s \sum_{t=1}^{\infty} \delta^t \widetilde{\mathbb{B}} \left(\frac{t}{a + T - t_1 + t}; k, r + x - 1 \right)
\end{aligned}
\tag{7}
$$

These future transactions depend on a number of different elements. The parameters r, a, s and b capture the distribution of order rates and baseline churn likelihoods across the population (e.g., for any randomly chosen member of the population, $E(\lambda) = r/a$ and $E(\theta) = s/b$). Through $P(\mathcal{A})$, customers with low t_x and high x might be more likely to have already become inactive, so there is a low probability of these individuals conducting transactions in the future. Customers with high x and high t_x are more likely to be alive, and to order often, so their DERT should be high.

Like all statistical models, this model is intended as a schematic of the actual data-generating process. To give the model some useful parametric structure, we treat the latent attrition process as a manifestation of a random variable. Though one can always propose more complicated versions of a model, such as allowing for duration dependence in purchase times or contagion across customers in their propensities to churn, we favor parsimony so as to avoid overparameterizing the model given certain limitations in a typical transactional dataset.

3 Empirical Analysis

The context in which we study the role of quality on customers' future transactions is that of an online market for freelance writing services. The firm in question operates a website on which customers can post orders for "jobs," and from which writers can claim jobs to complete. The types of jobs vary greatly. One example would be a 100-word description of a product that the customer, an online retailer, is selling on her website. Another is a 500-word summary of what participants at a conference might do for fun when exploring the host city. Orders include all of the information a writer would need to complete the job: the topic area (e.g., sports, health), intended audience, word count, and so forth. Customers are encouraged to be as specific as possible in their requirements, as that makes it more likely the customer will be satisfied with the results. In our taxonomy, we consider an order to be equivalent to the posting of a job.

Customers also choose a minimum rating, or grade, for the writers who are eligible to claim the order. The firm maintains a bank of reviewers who screen and rate the writers who register with the website. These reviewers are employed directly by the firm, and are considered to be experts in evaluating prose (many have Masters of Arts degrees, or similar qualifications). Upon initial application, a writer submits a writing sample, and a reviewer rates the writer as A, B, C or D. The firm's website provides examples of work from the different rating categories, so customers have a general idea about the differences to expect across the different ratings. Ratings differ according to objective criteria such as accuracy, grammar, style and vocabulary. A D-rated writer might produce work with errors and simple sentence structure with no creative insight, while work from an A-level writer will be of professional quality.

Customers pay, and writers earn, on a per-word basis, where the charge for each word depends on the rating in the order. The firm claims a fixed percentage of this fee, plus a small (less than a dollar) charge per order. Writers claim jobs from a list on a first-claim basis, so there is no bidding involved. Writers may claim jobs that are rated below their own ratings (e.g., an A-rated writer can choose a project from any level, but a B-rated writer cannot choose an A-rated order). In such cases, writers are paid the lower per-word fee. The company has told us that it has not experienced shortages of writers, with most job specifications being claimed within a day. Writers have another day to complete the job, and nearly all jobs are completed within 24 hours of posting.

Sometime after the writer returns the completed job to the customer, the firm's bank of reviewers assigns each job a grade. Customers are not involved in this grading process, and neither customers nor writers ever see the grade for a particular job. However, a writer's rating can be adjusted according to his grade history. This gives the writer an incentive to complete the job well; the grades determine if the writer's rating is adjusted up or down. The reviewers try to rate jobs as accurately and objectively as possible, as a way to ensure that customers receive the quality they pay for, and to reclassify writers as necessary. Writers can only be elevated to the A level manually, so the firm classifies all A-rated and B-rated jobs together in an A/B class. Reviewers may also assign a grade of E for completed jobs that do not meet even minimum standards.

3.1 Data summary

Our master dataset includes *all* completed jobs from the launch of the company in June 2008 to the end of our observation period at the end of July 2011. We are restricting our analysis to customers in either the United States or Canada whose first order takes place before the end of 2010, and to jobs for which the language is English. This dataset contains information on 24,059 completed jobs that were ordered by 3,048 distinct customers. For each job, we have identifiers for the customer and writer, the day that the order was placed, some other details of the job specification. We also have the requested rating of the job, as well as the grade the job received from the bank of reviewers. Table 1 shows the number of jobs requested at each quality rating, and the quality grade of the work that the writer delivered to the customer. By exploiting variation in the ratings, we can examine the impact of quality level delivered (assessed objectively by the reviewer), relative to the level that was requested by the customer, on customers' future transactional activity.

All observed transactions for a particular customer occur from the day of a customer's first order (t_1), until the end of our observation period (T). Treating the time of initial trial as the beginning of the customer relationship is consistent with prior research in customer base analysis (Schmittlein,

		Post-hoc quality grade				
		A/B	C	D	E	Total
	A	773	9	0	0	782
Requested	B	8784	614	6	1	9405
Rating	C	2050	5714	257	16	8037
	D	1668	3270	814	83	5835
	Total	13275	9607	1077	100	24059

Table 1: Number of jobs requested at each quality rating, and the quality grade of the work that the writer delivered to the customer.

Morrison, and Colombo 1987; Fader and Hardie 2001; Fader, Hardie, and Lee 2005a). If a customer places x orders during that observation period, his observed "frequency" is equal to $x/(T - t_1)$. A customer's "recency" is t_x, the week of the most recently observed order. Each day is represented as 1/7 of a week.

To control for the possibility that some of the firm's earlier adopters might behave differently than those customers whose first order came later, we divide the customer base into four cohorts based on the week of the first order. The 588 customers who placed their first order during the first 33 weeks of our data are considered to be in the first cohort. The 568 customers who placed their first order between weeks 33 and 66 are assigned to the second cohort. The 911 customers placing their first orders between weeks 66 and 99 are assigned to the third cohort, and the 981 customers placing their first orders between weeks 99 and 130 are assigned to the fourth cohort.

3.2 Model estimation

In this example, the transaction attributes represent the requested and delivered quality of the jobs. Although there are many functional forms that we could choose, we consider models of the form $\log q_k = \beta' z_k$, where z_k is a vector of job-specific covariates and β is a vector of coefficients. Effects that increase q_k increase the probability of churn.

The elements of z_k include the following:

- z_{first}: a indicator of the customer's first job;
- z_{coh2}, z_{coh3}, z_{coh4}: indicators for time-invariant cohort effects;

- z_{Ak}, z_{Ck}, z_{Dk}: indicators for the requested quality level of job k;

- z_{Lk}, z_{Hk}: indicators for whether job k was lower (L) or higher (H) than the requested service level; and

- z_{LD}, z_{HD}, z_{LC}: interactions among requested and delivered quality ratings;

All coefficients, except those on z_{Lk} and z_{Hk}, are stationary. The coefficients β_{Lk} and β_{Hk} represent the effect on the churn probability from missing or exceeding the specifications of job k. These effects can change from job to job, according to the customer's recent experience. To capture how these sensitivities change, we define a set of six η parameters that affect β_{Lk} and β_{Hk} in the following ways:

$$\beta_{L,k+1} = \beta_{Lk} + \eta_L + \eta_{LL} z_{Lk} + \eta_{HL} z_{Hk} \tag{8}$$

$$\beta_{H,k+1} = \beta_{Hk} + \eta_H + \eta_{LH} z_{Lk} + \eta_{HH} z_{Hk} \tag{9}$$

The coefficient β_{Lk} changes by η_L regardless of the rating given to job k, capturing a drift in customers' sensitivity to receiving a lower-than-requested service level. The terms η_{LL} and η_{HL} capture the extent to which customers' sensitivity to receiving a lower-than-requested service level on job $k+1$ is affected by receiving a lower (η_{LL}) or higher (η_{HL}) level of service than was requested for job k. The coefficient β_{Hk} evolves in a similar manner. By allowing β to be dynamic in this way, we allow for customers' responses to service quality to be affected by a customer's experience (Bolton 1998). For example, if η_{LL} were positive, then after having a bad experience with the firm, a customer would be even more sensitive to subsequent bad experiences.

To assess the role of service quality on churn propensities, we tested three variants of the model. Model 3 is the full model as described. Model 1 is a "baseline" model that ignores all service quality effects. Model 2 is similar to Model 3, with all of the insignificant η parameters removed.

Table 2 contains descriptions of the model parameters, along with maximum likelihood estimates and standard errors. The subscripts for the elements of β in the table correspond to those of z. The most interesting estimates are those on β_{L1} and η_{LL}, which are both positive. This result suggests

	Model 1		Model 2		Model 3		Description
	est	se	est	se	est	se	
r	0.90	0.04	0.90	0.04	0.90	0.04	shape parameter on λ
a	0.77	0.04	0.77	0.04	0.77	0.04	scale parameter on λ
s	1.09	0.07	1.08	0.07	1.10	0.07	shape parameter on θ
b	1.13	0.19	1.14	0.19	1.17	0.20	scale parameter on θ
β_{first}	-0.22	0.07	-0.22	0.07	-0.21	0.07	effect of customer's first job
β_{coh2}	-1.15	0.11	-1.16	0.11	-1.15	0.11	fixed effect for cohort 2
β_{coh3}	-1.00	0.10	-1.01	0.11	-1.00	0.11	fixed effect for cohort 3
β_{coh4}	-0.81	0.11	-0.81	0.11	-0.80	0.11	fixed effect for cohort 4
β_A	0.88	0.11	0.89	0.11	0.89	0.11	effect for requested quality level A
β_C	-0.30	0.06	-0.27	0.07	-0.27	0.07	effect for requested quality level C
β_D	-0.51	0.08	-0.36	0.15	-0.37	0.15	effect for requested quality level D
β_{L1}			0.19	0.13	0.21	0.14	effect for rating being "lower" than requested
β_{H1}			0.01	0.09	0.01	0.10	effect for rating being "higher" than requested
β_{LD}			0.21	0.36	0.24	0.38	interaction effect between "lower" and requested level D
β_{HD}			-0.18	0.17	-0.15	0.17	interaction effect between "higher" and requested level D
β_{LC}			-0.53	0.26	-0.52	0.26	interaction effect between "higher" and requested level C
η_L					-0.01	0.01	evolution parameter on β_L
η_H					0.00	0.01	evolution parameter on β_H
η_{LL}			0.18	0.12	0.29	0.21	evolution parameter on β_L after a "lower" rating
η_{LH}					0.05	0.13	evolution parameter on β_H after a "lower" rating
η_{HL}					0.01	0.04	evolution parameter on β_L after a "higher" rating
η_{HH}					-0.01	0.02	evolution parameter on β_H after a "higher" rating

Table 2: Parameter estimates

that, as expected, missing the requested level of service for the first job increases the probability of churn. It also suggests that the magnitude of that effect will be larger for the next job with a missed service level. Thus, the churn probabilities increase across jobs when customers repeatedly receive lower-than-requested service.

We also see that β_{H1}, and the associated η parameters, are not significantly different from zero. This asymmetry in the effect of service quality on customers' tendency to churn is consistent with losses looming larger than gains (Kahneman and Tversky 1979; Hardie, Johnson, and Fader 1993). Our findings are also in line with prior research by Bolton (1998), who found that perceived losses adversely impact the duration of a customer's relationship in a contractual setting while perceived gains did not have a significant impact on the duration of the relationship.

3.3 Model assessment

We compare the performance of the three model specifications using a series of assessments. A likelihood ratio test suggests a weak preference for Model 2 over Model 1 ($\chi^2_6 = 11.0, p = .088$); we cannot infer a preference for Model 3 over Model 2 ($\chi^2_5 = 1.29, p = .936$), or for Model 3 over Model 1 ($\chi^2_9 = 10.1, p = .342$). One shortcoming of relying only on the likelihood ratio test, however, is that it does not consider the extent to which the incorporation of transaction attributes improves forecasting performance. Thus, in addition to the likelihood ratio test, we compare model performance using two additional measures of fit.

We calculated the mean absolute percentage error (MAPE) associated with the models' prediction of the weekly number of repeat orders made by customers in our sample and the root mean squared error (RMSE) for the models' predictions of the distribution of the number of orders made by the sample. We find that the MAPE is similar across model specifications, with Models 2 and 3 having slightly lower errors compared to Model 1. In terms of the RMSE, we again find evidence to suggest that the transaction attributes contribute to model fit. Models 2 and 3 have lower RMSEs compared to Model 1 for the data used to estimate the model during the calibration and forecasting periods. Using a holdout sample for cross-validation reveals that, while Model 2 has a lower RMSE than Model 1, Model 3 has a higher RMSE during the forecasting period which would suggest that the model is overparameterized.

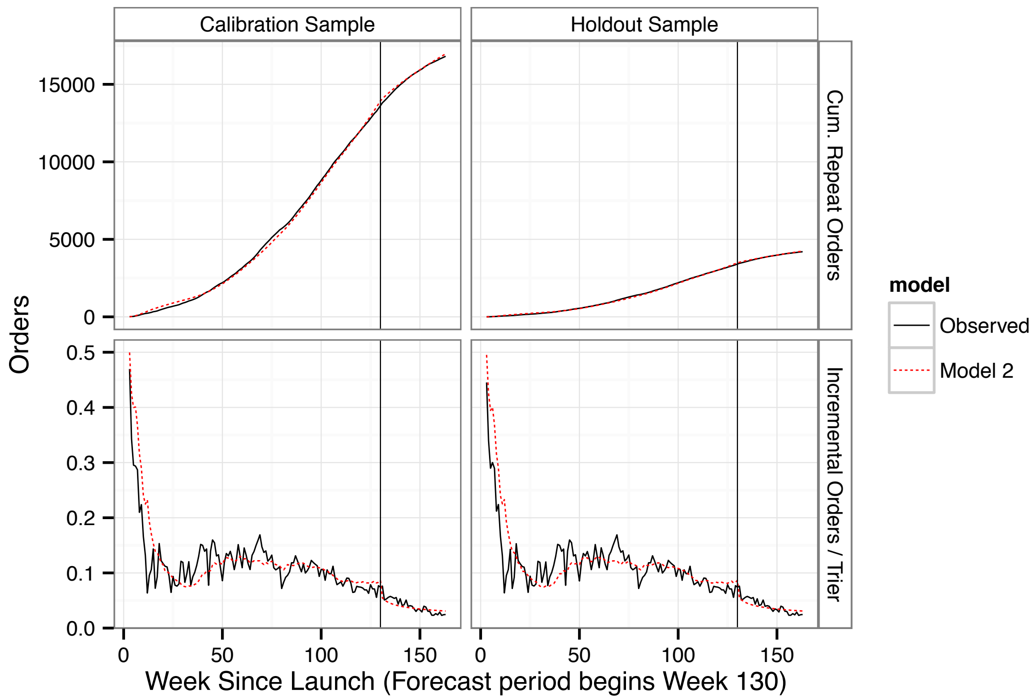
Details of these posterior predictive tests are in Appendix C, which as available as part of the online supplement. Taken together with the likelihood ratio test, we believe that the posterior predictive tests provide evidence that transaction attributes improve model performance and contribute to forecasting accuracy. Based on these analyses, we focus on the results using Model 2 for the remainder of our discussion.

To provide a better sense for how well the proposed model captures customers' observed behavior, the panels in Figure 1 illustrate model fit at the aggregate level. Figure 1a plots the cumulative and incremental number of weekly orders, for both in-sample and holdout populations. The vertical lines at Week 130 divide the calibration and forecast time periods. We used only data to the left of the lines for estimating the model parameters, and we included only those customers whose initial order was before Week 130. Model 2 does well in tracking the number of orders from week to week. In Figure 1b, we compare the histogram of pre-customer order counts with the distribution of counts from Model 2 predicts. Again, Model 2 appears to fit rather well.

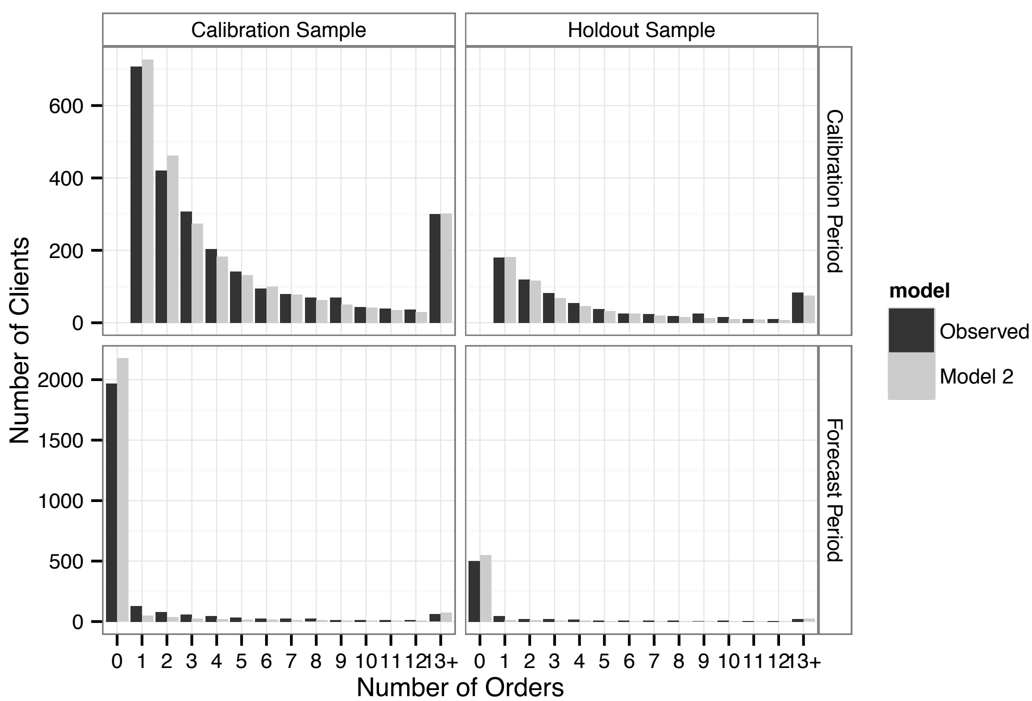
At the level of an individual customer, one managerially relevant test statistic is the probability that a customer will place an order sometime in the future. While the probability of being active, $P(\mathcal{A})$, is a commonly used construct in customer base analysis, we cannot use it as a model checking tool because we cannot observe the customer's activity state directly. Instead, the appropriate metric is $P^*(0) = P(X^*(t^*) = 0|x, t_x, \cdot)$, the posterior probability that a customer will place no orders during a forecast period. We test how well Model 2 predicts which customers will order during the forecast period using a calibration plot. First, we assign each customer to one of 15 "bins", according to the customer's posterior $P^*(0)$. A customer is assigned to bin i if $(i-1)/15 < P^*(0) \leq i/15$ for $i = 1 \ldots 15$. Next, we compute the observed proportion of customers in each bin who do not place an order during the forecast period. We consider the model to be well-calibrated if the predicted probabilities and observed proportions are aligned. Figure 2 confirms that they are. Each dot represents the membership of the bin. The x-coordinate is the midpoint of the bin, and the y-coordinate is the observed incidence of "no orders" for the members of that bin. "Perfect" calibration would have occurred if all of the dots fell exactly on the 45° line. Of course, we expect some random variation around this line, so we can still be confident that Model 2 forecasts the incidence of future orders, at the customer level, quite well.

3.4 Forecasting quality data

In Section 2.1 we discussed the need to model the sequences of covariates for the purpose of estimating conditional expectations and DERT rather than simply plugging in a fixed value. The specifics of such a model depend on the context. For this dataset, there are two sources of variation in the covariates: the service level that a customer requested and the level that was delivered. For the "requested" model, we assume that each customer has a latent, stationary probability of placing A, B,



(a) Weekly incremental repeat orders per previously acquired customer. The vertical line is at Week 130, the end of the calibration period and the start of the forecast period.



(b) Observed and predicted histograms of orders.

Figure 1: Fit and forecast assessment for Model 2.

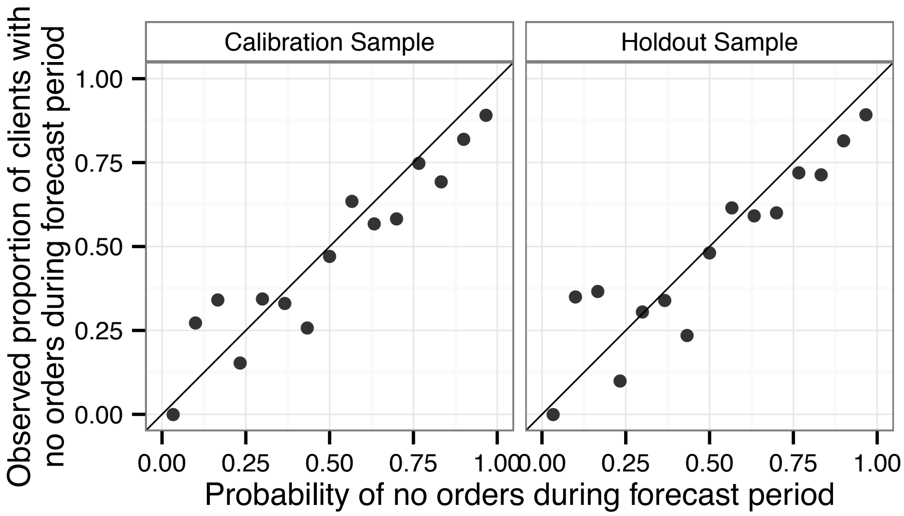


Figure 2: Predicted vs observed probability of a particular customer making no orders during the forecast period.

C or D-rated orders. This probability vector is heterogeneous and varies across customers according to a Dirichlet distribution, allowing some customers to always choose the same rating, while other customers vary more in their choices. Under the further assumption of a zero-order choice process, we can infer the likelihood of a particular order pattern by using the maximum likelihood estimates of a Dirichlet-multinomial mixture model.

The Dirichlet-multinomial parameters for this dataset are .04, .26, .22 and .12 for ordered ratings A, B, C and D, respectively. These low values (all less than one) suggest high polarization in the Dirichlet mixing distribution; even though there may be variation across customers, a single customer is likely to place orders for the same level of quality across the jobs he orders. To simulate hypothetical orders, we sample a probability vector from each customer's posterior Dirichlet distribution. Since most customers order at the same rating level every time, these posterior probabilities are even more concentrated on a single quality level compared to the choice probabilities of the prior distribution. We then use the empirical distribution of the job grades for each rating level to get the service level delivered for each simulated job. In general, we find that there is little variation in B_k across simulated sequences of z_k.

4 How Transactional Patterns Affect DERT and Incremental DERT

Using the parameter estimates of Model 2, we can examine how information from transaction attributes affects expectations of the future transactional activity of heterogeneous customers. Specifically, we examine how falling short of the expected level of service affects DERT, for customers with different frequency (x) and recency (t_x) data. Figure 3 plots the contours that connect the same levels of DERT at $T = 130$ for hypothetical customers who placed the first order at time $t_1 = 1$, and who requested quality grades B, C or D for the most recent order. For each of requested service levels, we consider delivered service levels that are lower than, or the same as, what was requested for that most recent order. We assume that the level of service delivered was the same as what was requested for all other orders placed by the customer. These iso-value curves are similar in spirit to those introduced by Fader, Hardie, and Lee (2005b) for the Pareto/NBD model. Since DERT is a posterior expectation based affected by the likelihood that a customer remains active, as anticipated, we see "backward-bending" contours (Fader, Hardie, and Lee 2005b). When the number of orders is large and the most recent order was in the distant past, it is more likely that the customer has already become inactive than if the x^{th} order was made more recently.

The iso-DERT curves in Figure 3 reveal the relationship between customers' transaction histories and expected future activity, but they do not show the *incremental* effect of missing the requested service level. To assess the incremental impact of deviations in the level of service on expected future transactions, we calculate the difference in DERT between what we would expect when the most recent job is rated at the same level of service the customer ordered and what we would expect if the level of service delivered was lower. We refer to this difference as "incremental DERT." This metric offers a long-term assessment of the marketing investment's impact, and forms an upper bound on the amount the firm should invest. In transactions for which incremental DERT is small, it may not be worth the firm's effort to monitor and evaluate the service encounters. Those transactions for which the incremental DERT is large, however, may warrant additional resources to ensure the

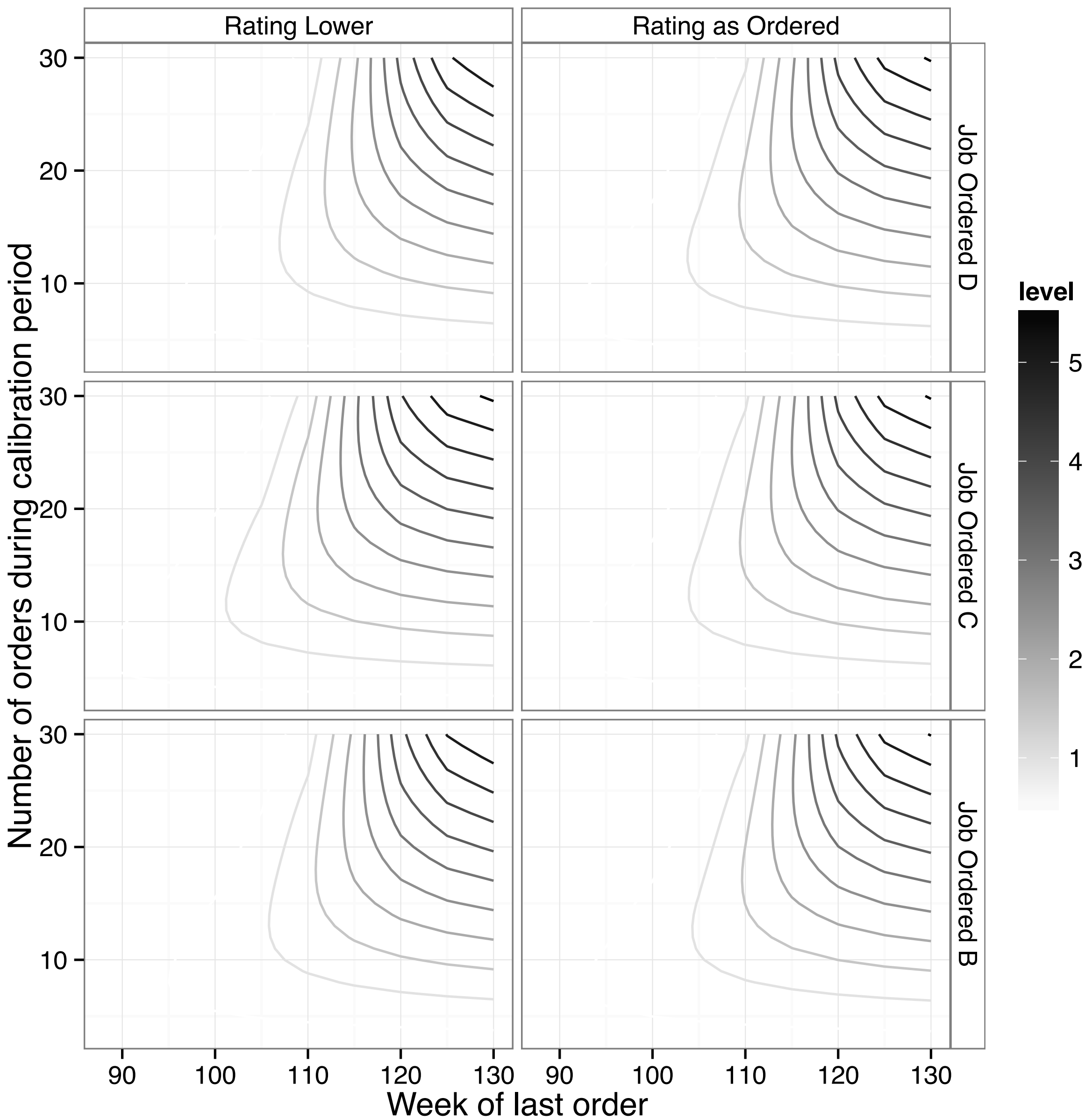


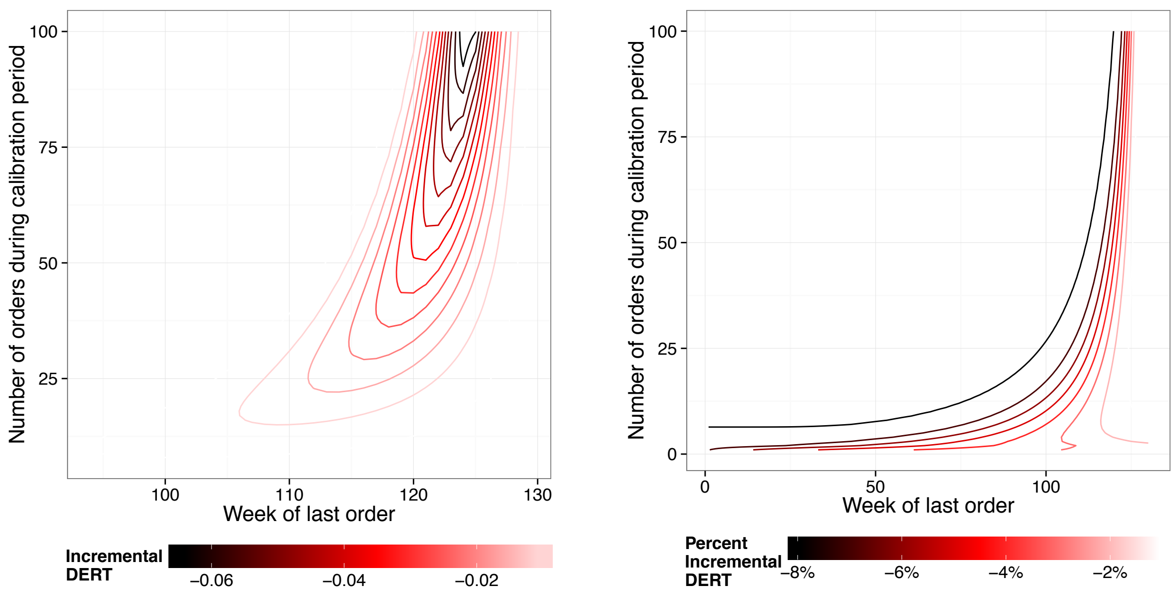
Figure 3: DERT iso-value curves for hypothetical customers whose first order came at $t_1 = 1$. The panels condition on the ordered job quality, and whether the last job was rated lower or the same than what was ordered. The x axis of each panel is the week number of the most recent order ($T = 130$), and the y axis is the number of observed orders.

appropriate level of service is delivered. In our empirical context, incremental DERT can be viewed as the penalty associated with delivering a level of service lower than what a customer requested.

The expected cost of missing a requested service level for a brand new customer is the incremental DERT when $x = 1$ and $t_1 = t_x = T$. That is, immediately after the first job is ordered and a new customer comes under observation, incremental DERT is the number of discounted future transactions that we expect to be lost if the delivered service level were below what was requested. For this particular dataset, the percentage change is 2.9% for new customers who ordered a B-level job, 2.5% for a C-level job, and 2.2% for a D-level job. For existing customers, incremental DERT depends crucially on how many orders the customer has placed and how long ago his last order was made. Figure 4 shows this relationship in the form of "iso-incremental DERT" curves for a customer who has requested a service level of B, in both absolute and percentage terms, for $t_1 = 1$ and $T = 130$. The levels of these curves represent the incremental DERT, evaluated at varying levels of frequency and recency. The darker contours represent differences that are *more negative*, for which the mismatch in service levels has more of an effect.[4]

In absolute terms, at a given level of frequency, falling short of the requested service level appears to have the greatest effect when recency is neither too high nor too low. Take the case of a customer who has conducted a given number of transactions. If the last transaction was conducted recently, it is likely that the customer is still active. Regardless of whether the service delivered was at or below the level requested, the recency of the transaction leads us to estimate a high DERT for that customer. Alternatively, if the order were placed at a time in the distant past, the customer is more likely to have already defected, regardless of the service quality level, so we estimate a low DERT. In both of these cases, the difference in DERT between the case in which the service delivered matches what was requested and the case in which it is below that which was requested will be small compared to the incremental DERT for a customer with moderate recency. For those customers, there is more uncertainty about whether the customer is still active. Small differences in attrition probabilities will yield more substantial decreases in DERT when the requested service

[4]We observe a similar pattern as depicted in Figure 4 for jobs that exceed the order specifications, except with positive differences in DERT. However, for this dataset, the magnitude of the effect is very small.



(a) Absolute difference

(b) Percent difference

Figure 4: Iso-incremental DERT curves when missing the requested service level. Observed recency is on the x-axis and frequency is on the y-axis. Each curve connects points for which the incremental DERT is the same.

level is missed. We see that the range of incremental DERT is the widest at low or moderate order frequencies.

Figure 4b illustrates the DERT contours in percentage terms. As shown in Figure 4a, for a given number of orders x, there are two values t_x that correspond to the same absolute difference in DERT. As DERT increases with recency for a given value of x (as shown in Figure 3), incremental DERT on a percentage basis will be smaller at higher values of recency.

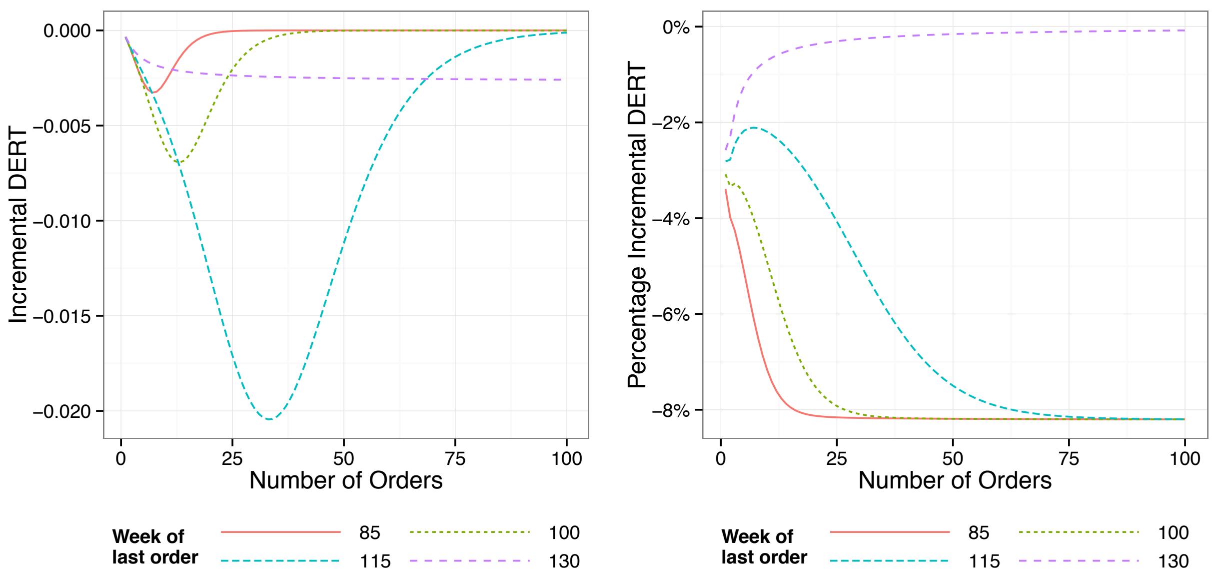
Figure 4 demonstrates that the expected difference in a customer's remaining value between meeting and falling short of the requested service level depends on both the recency and frequency of the customer's transactions. When the goal of CRM is to identify those customers for whom targeted marketing actions generate the greatest financial impact, our findings should caution against scoring customers solely on popular measures like P(Alive) or CLV. While the most recent and frequently transacting customers are likely to be the most valuable, the effect of transaction attributes is not much different from customers who last purchased a long time ago.

To provide another perspective on customers' remaining value, Figure 5 shows incremental

DERT (absolute in Figure 5a, and percent in Figure 5b) as a function of frequency, for select values of recency. That is, each curve corresponds to a vertical slice of either Figure 4a or 4b. For transaction histories with lower recency (e.g., $t_x = 85$ and $t_x = 100$), we observe an initial increase in the magnitude of the incremental DERT (in absolute terms) as the number of orders increases. But, with a very large number of orders, the magnitude of incremental DERT decreases and approaches zero. This pattern reveals the interplay of two factors. First, the large number of orders is indicative of a high transaction rate (λ). For these customers, missing the requested service level puts a lucrative revenue stream at risk. But, with a large number of orders, the low recency of the last order suggests that the customer may have already become inactive. Given that the increased likelihood that the customer has already lapsed, the increased cost associated with missing the requested service level is lower, resulting in the reduced magnitude of incremental DERT. At the highest level of recency ($t_x = 130$), the customer is known to be still active, and DERT increases with the number of orders. We thus observe that the magnitude of incremental DERT increases with frequency, but at a much slower rate.

Figure 5b presents a similar analysis in percentage terms. At the highest level of recency ($t_x = 130$), when the customer has just conducted a transaction and is known to still be active, DERT increases with frequency. Therefore, incremental DERT, while relatively flat in absolute terms, falls when expressed as a percentage of DERT. For the remaining three levels of recency, higher transaction frequency ultimately results in a higher magnitude of incremental DERT as a percentage. As the number of transactions increases along these three curves, so too does the likelihood that these customers have already become inactive, thereby driving down DERT. With incremental DERT (in absolute terms) being compared to a smaller base, the incremental DERT increases as a percentage.

Prior research has suggested that the value of CRM tools lies in their ability to facilitate learning about customers (Mithas, Krishnan, and Fornell 2005), and that this value is limited based on the quality of information the firm has available about its customers (Homburg, Droll, and Totzek 2008). Our findings reveal that differences in customer value that are associated with variation in transaction attributes (e.g., the level of service delivered on a transaction) depend on customers' past transaction



(a) Absolute difference

(b) Percent difference

Figure 5: Absolute and percent incremental DERT as a function of frequency, for different levels of recency.

activity. If a customer is more obviously active or inactive deviations from the requested service level provide little information on the customer's DERT. It is when there is increased uncertainty as to whether a customer is active or inactive that the level of service delivered relative to that which was requested allows us to learn more about a customer's DERT. To the best of our knowledge, our research is the first to explore the value of transaction attributes using the latent attrition models frequently employed in customer base analysis and customer valuation, and how past activity affects the differences in customer value associated with transaction attributes.

5 Discussion

We present a flexible latent attrition model that incorporates transaction attributes in customer base analysis, and describe an empirical application that defines those attributes as indicators for relative service quality. The model allows us to derive "incremental DERT," a metric of the discounted expected return on changes in those attributes. For transactions that are about to happen immediately, incremental DERT can serve as an upper bound on the amount a firm should invest to change one of those attributes. We also describe patterns in the relationship between incremental DERT and the recency/frequency profile of the customer. Understanding these patterns and quantifying the effects allow a manager to more accurately estimate DERT for all members of the customer base. Model parameters can be estimated using standard maximum likelihood methods and DERT can be computed as a truncated summation.

As firms use customer base analysis and customer valuation models to score and rank customers according to the value they hold for a firm (Wuebben and von Wangenheim 2008), managers should be interested in tracking and compiling transaction attributes that may be informative of customers' future value. Acquiring such information, however, often entails a cost. For example, firms may incur costs to monitor their salesforce to ensure compliance with required activities (John and Weitz 1989). A simple heuristic to identify those transactions in which firms should be willing to invest more in monitoring could be customers' purchase frequency. After all, such customers are likely to be among the most valuable customers to a firm. Our results, however, suggest that for such customers, the difference in discounted long-term value to the firm between meeting and falling short of the requested service level (reflected by incremental DERT) is low for those customers. For customers who have not conducted as many transactions, our estimates of their value to the firm are more sensitive to information on the level of service delivered. It is these customers for whom estimates of long-term value are most subject to change if the level of service in a transaction comes up short of specifications.

In our empirical context, we see that the magnitude of the effect of missing specified levels of service is larger than exceeding specified levels. This is consistent with the idea of losses looming larger than gains. While extant research has questioned the wisdom of trying to delight customers by exceeding their expectations, due to the possibility of raising customers' future expectations (Rust and Oliver 2000), our analysis suggests that coming up short of the service customers have requested poses a greater risk to a customer's continued relationship with the firm and the firm's ability to capture the corresponding revenue stream.

Using our framework as a foundation, there are a number of promising directions with which research could continue. While we focus our attention on customer retention and the value of existing customers, a similar modeling approach could be employed to jointly investigate customer acquisition and retention (Schweidel, Fader, and Bradlow 2008; Musalem and Joshi 2009). Doing so could provide firms with guidance for how to balance marketing expenditures across the two activities (Reinartz, Thomas, and Kumar 2005). Another area to explore is the effect of strategic investments in service quality, a practice that the firm that supplied our data did not employ. While it may be costly for a firm to invest in improving service encounters and monitoring these encounters for all customers, the firm may have resources to focus on select customers. The firm could use incremental DERT as a criterion for selecting those target customers. If the costs of delivering better than expected service experiences are the same across customers, such an allocation rule would be equivalent to putting your money where it will deliver the most “bang for the buck.” These actions would be consistent with the management principle of “return on quality.” (Rust, Zahorik, and Keiningham 1995). In many cases, we would need to account for the firm’s actions when estimating the effect on retention probabilities (Manchanda, Rossi, and Chintagunta 2004; Schweidel and Knox 2013). To alleviate such concerns, one might choose to proceed in this research area with a carefully designed field experiment.

Another area for future research would be to develop further means of accounting for variation in the effect of customer-firm touch points on future customer behavior. While we rely on observable transaction attributes (i.e., the requested and evaluated service levels) as a means of evaluating the level of service, future work may explore if such a measure could be inferred with additional information about the transaction. For example, in our empirical context, if the transaction data indicated the writer who completed each job, the firm may be able to evaluate the “quality” of each writer based on the effect they have on customer churn. Another example of a firm that offers a marketplace is eBay, which connects buyers and sellers. Evaluating sellers based on the churn of buyers who have recently interacted with them may provide an indication of which sellers should be rewarded versus which sellers are potentially costing the firm business.

A cost of conducting such an analysis is the detail in the data that must be collected. While the early latent attrition models that appeared in the marketing literature relied on recency and frequency as sufficient statistics, as in our analysis, recognizing the variation in customer tendencies that exist across transactions require data be tracked at the transaction level. It is an empirical question as to extent to which incorporating such sources of variation into the analysis will affect managerial decisions. As the answer to this question may vary from context to context, additional research across a range of empirical applications is warranted, recognizing the costs associated with acquiring and analyzing data on customer-firm interactions.

Appendices

A Derivations

In this section, we use the definitions and symbols that are defined in Table 3.

$\Gamma(k) = \int_0^\infty t^{k-1}e^{-t}dt$	Gamma function
$\gamma(k,\lambda) = \int_0^\lambda t^{k-1}e^{-t}dt$	Lower incomplete gamma function
$\mathcal{G}_z(r,a) = \gamma(r,az)/\Gamma(r)$	cdf of a gamma distribution with shape r and rate a
$d\mathcal{G}_z(r,a) = \dfrac{a^r}{\Gamma(r)}z^{r-1}e^{-az}$	Density of a gamma distribution with shape r and rate a
$\mathbb{B}(k,r) = \int_0^1 u^{k-1}(1-u)^{r-1}du$	Beta function
$\mathbb{B}(z;k,r) = \int_0^z u^{k-1}(1-u)^{r-1}du$	Incomplete beta function
$\widetilde{\mathbb{B}}(z;k,r) = \mathbb{B}(z;k,r)/\mathbb{B}(k,r)$	Regularized incomplete beta function (equivalent to cdf of beta distribution with parameters k and r, evaluated at z)
${}_2F_1(a,b,c;z)$	Gaussian hypergeometric function
$P(\mathcal{A}\vert\lambda,\theta)$, $P(\mathcal{A})$	Conditional and marginal probabilities that a customer has not yet churned by time T.

Table 3: Definitions of symbols and functions used in the paper.

To derive the marginal likelihood in Equation 3, we integrate the individual-level data likelihood in Equation 1 over two gamma densities, one for λ and one for θ.

$$
\begin{aligned}
\mathcal{L} &= \int_0^\infty \int_0^\infty f(x, t_{2:x}|\lambda, \theta, q_{1:x}) d\mathcal{G}_\lambda(r, a) d\mathcal{G}_\theta(s, b) \\
&= \int_0^\infty \int_0^\infty \lambda^{x-1} e^{-\lambda(t_x - t_1) - \theta B_{x-1}} \left[1 - e^{-\theta q_x}\left(1 - e^{-\lambda(T - t_x)}\right)\right] \frac{a^r}{\Gamma(r)} \lambda^{r-1} e^{-a\lambda} \frac{b^s}{\Gamma(s)} \theta^{s-1} e^{-b\theta} d\lambda d\theta \\
&= \frac{a^r}{\Gamma(r)} \frac{b^s}{\Gamma(s)} \int_0^\infty \lambda^{r+x-2} e^{-\lambda(a+t_x-t_1)} d\lambda \int_0^\infty \theta^{s-1} e^{-\theta(b+B_{x-1})} d\theta \\
&\quad - \frac{a^r}{\Gamma(r)} \frac{b^s}{\Gamma(s)} \int_0^\infty \lambda^{r+x-2} e^{-\lambda(a+t_x-t_1)} d\lambda \int_0^\infty \theta^{s-1} e^{-\theta(b+B_x)} d\theta \\
&\quad + \frac{a^r}{\Gamma(r)} \frac{b^s}{\Gamma(s)} \int_0^\infty \lambda^{r+x-2} e^{-\lambda(a+T-t_1)} d\lambda \int_0^\infty \theta^{s-1} e^{-\theta(b+B_x)} d\theta \\
&= \frac{\Gamma(r+x-1)}{\Gamma(r)} \frac{a^r}{(a+t_x-t_1)^{r+x-1}} \left(\frac{b}{b+B_{x-1}}\right)^s \left[1 - \left(\frac{b+B_{x-1}}{b+B_x}\right)^s \left(1 - \left(\frac{a+t_x-t_1}{a+T-t_1}\right)^{r+x-1}\right)\right]
\end{aligned}
\tag{10}
$$

By rearranging terms, we can write the marginal likelihood equivalently as

$$
\mathcal{L} = \frac{\Gamma(r+x-1)}{\Gamma(r)} \frac{a^r}{(a+T-t_1)^{r+x-1}} \left(\frac{b}{b+B_x}\right)^s \left[1 - \left(\frac{a+T-t_1}{a+t_x-t_1}\right)^{r+x-1} \left(1 - \left(\frac{b+B_x}{b+B_{x-1}}\right)^s\right)\right] \tag{11}
$$

We can also compute the expected number of orders for any randomly chosen customer in the population. Our approach draws inspiration from Section 4.3 in Fader, Hardie, and Lee (2005a). Let τ be the time of job immediately after which the customer churns. Therefore, conditional on k, the probability that τ is sometime after t is equal to the probability of surviving all k transactions that occurred before t . This survival probability is $e^{-\theta B_k}$. The probability of making k transactions is a shifted Poisson (since k starts at 1). By summing over all possible values of k,

$$
P(\tau > t) = \sum_{k=1}^{\infty} \frac{(\lambda t)^{k-1} e^{-\lambda t}}{(k-1)!} e^{-\theta B_k} \tag{12}
$$

By differentiating Equation 12 with respect to t, we get the pdf of τ.

$$
\begin{aligned}
g(\tau) &= \sum_{k=1}^{\infty} \frac{e^{-\theta B_k} \lambda^{k-1}}{(k-1)!} \frac{d}{dt} \left[t^{k-1} e^{-\lambda t} \right] \\
&= e^{-\lambda \tau} \sum_{k=1}^{\infty} \frac{e^{-\theta B_k} (\lambda \tau)^{k-1}}{(k-1)!} \left[\lambda - \frac{k-1}{\tau} \right] \qquad (13)
\end{aligned}
$$

To get the expected number of transactions for an individual customer, we have to consider two cases. In the first case, the customer survives until time t, so the expected number of repeat orders (not including the first order) is λt, times the probability that $\tau > t$. In the second case, the customer dies at time τ, which is sometime before t. In this case, the expected number of repeat transactions is $\lambda \tau$. Since τ is unknown, we can get the expected number of repeat transactions by integrating τ over the entire interval in question, with respect to $g(\tau)$.

$$
\begin{aligned}
E[X(t)|\lambda, \theta] &= \lambda t \left[\sum_{k=1}^{\infty} \frac{(\lambda t)^{k-1} e^{-\lambda t}}{(k-1)!} e^{-\theta B_k} \right] + \lambda \int_0^t \tau e^{-\lambda \tau} \sum_{k=1}^{\infty} \frac{e^{-\theta B_k} (\lambda \tau)^{k-1}}{(k-1)!} \left[\lambda - \frac{k-1}{\tau} \right] d\tau \\
&= \lambda t \left[\sum_{k=1}^{\infty} \frac{(\lambda t)^{k-1} e^{-\lambda t}}{(k-1)!} e^{-\theta B_k} \right] + \sum_{k=1}^{\infty} \frac{\lambda^k e^{-\theta B_k}}{(k-1)!} \int_0^t e^{-\lambda \tau} \tau^k \left[\lambda - \frac{k-1}{\tau} \right] d\tau \\
&= \left[\sum_{k=1}^{\infty} \frac{(\lambda t)^k e^{-\lambda t}}{(k-1)!} e^{-\theta B_k} \right] + \sum_{k=1}^{\infty} \frac{e^{-\theta B_k}}{(k-1)!} \left[\gamma(k, \lambda t) - (\lambda t)^k e^{-\lambda t} \right] \\
&= \sum_{k=1}^{\infty} \frac{\gamma(k, \lambda t)}{\Gamma(k)} e^{-\theta B_k} \qquad (14)
\end{aligned}
$$

To get the prior expectation for a randomly-chosen member of the population (Equation 4), we integrate Equation 14 over the prior densities of λ and θ.

$$
\begin{aligned}
E[X(t)] &= \sum_{k=1}^{\infty} \frac{a^r}{\Gamma(r)} \frac{b^s}{\Gamma(s)} \frac{1}{\Gamma(k)} \int_0^{\infty} \gamma(k, \lambda t) \lambda^{r-1} e^{-a\lambda} \int_0^{\infty} \theta^{s-1} e^{-\theta(B_k + b)} d\theta \, d\lambda \\
&= \sum_{k=1}^{\infty} \frac{a^r}{\Gamma(r)} \frac{1}{\Gamma(k)} \left(\frac{b}{b + B_k} \right)^s \int_0^{\infty} \gamma(k, \lambda t) \lambda^{r-1} e^{-\alpha \lambda} d\lambda \qquad (15)
\end{aligned}
$$

To solve the last integral in Equation 15, we apply Equation 6.455.1 in Gradshteyn and Ryzhik

(2000), which expresses the integral in terms of a Gaussian hypergeometric function.

$$E[X(t)] = \sum_{k=1}^{\infty} \frac{a^r}{\Gamma(r)} \frac{1}{\Gamma(k)} \left(\frac{b}{b+B_k}\right)^s \frac{t^k \Gamma(r+k)}{k(a+t)^{r+k}} \, {}_2F_1\left(1, r+k; k+1; \frac{t}{a+t}\right) \tag{16}$$

We can simplify this expression using identities in two sections of the NIST Handbook of Mathematical Functions (Olver et al. 2010). First, we transform the hypergeometric function using the identity in Section 15.8.1. Then, we apply the hypergeometric representation of an incomplete beta function from Section 8.17.9. Finally, we regularize the incomplete beta function to get Equation 4.

$$\begin{aligned} E[X(t)] &= \sum_{k=1}^{\infty} \frac{a^r}{\Gamma(r)} \frac{1}{\Gamma(k)} \left(\frac{b}{b+B_k}\right)^s \frac{t^k \Gamma(r+k)}{k(a+t)^{r+k}} \left(\frac{a+t}{a}\right)^r \, {}_2F_1\left(k, 1-r; k+1; \frac{t}{a+t}\right) \\ &= \sum_{k=1}^{\infty} \frac{\Gamma(r+k)}{\Gamma(r)\Gamma(k)} \left(\frac{b}{b+B_k}\right)^s \mathbb{B}\left(\frac{t}{a+t}; k, r\right) \\ &= \sum_{k=1}^{\infty} \left(\frac{b}{b+B_k}\right)^s \widetilde{\mathbb{B}}\left(\frac{t}{a+t}; k, r\right) \end{aligned} \tag{17}$$

Note that the regularized incomplete beta function is equivalent to the cdf of a beta distribution.

Given a customer's transaction history, we can derive the joint posterior density of λ and θ by applying Bayes' Theorem.

$$\begin{aligned} g(\lambda, \theta | x, t_1 \ldots t_x) &= \frac{1}{\mathcal{L}} f(x, t_{2:x} | \lambda, \theta) d\mathcal{G}_\lambda(r, a) \, d\mathcal{G}_\theta(s, b) \\ &= \frac{1}{\mathcal{L}} \frac{a^r b^s}{\Gamma(r)\Gamma(s)} \lambda^{r+x-2} e^{-\lambda(a+t_x-t_1)} \theta^{s-1} e^{-\theta(b+B_{x-1})} \left[1 - e^{-\theta q_x}\left(1 - e^{-\lambda(T-t_x)}\right)\right] \\ &= d\mathcal{G}_\lambda(r+x-1, a+t_x-t_1) \, d\mathcal{G}_\theta(s, b+B_{x-1}) \\ &\quad \times \frac{1 - e^{-\theta q_x}\left(1 - e^{-\lambda(T-t_x)}\right)}{1 - \left(\frac{b+B_{x-1}}{b+B_x}\right)^s \left[1 - \left(\frac{a+t_x-t_1}{a+T-t_1}\right)^{r+x-1}\right]} \end{aligned} \tag{18}$$

One important quantity of interest is the probability that a customer is still "alive" at the end of the observation period. At time T, the customer is in one of two possible states. One state is that after the x^{th} transaction, the customer churned. This occurs with probability p_x. The other state is that the customer survived the last transaction, but has not purchased since. This occurs with probability $(1 - p_x)e^{-\lambda(T-t_x)}$. Therefore, the probability of being alive at time T, conditional on purchase history, is

$$P(\mathcal{A}|\lambda,\theta) = \frac{(1-p_x)e^{-\lambda(T-t_x)}}{p_x + (1-p_x)e^{-\lambda(T-t_x)}} = \frac{e^{-\theta q_x - \lambda(T-t_x)}}{1 - e^{-\theta q_x}\left(1 - e^{-\lambda(T-t_x)}\right)} \tag{19}$$

Integrating Equation 19 across the posterior density in Equation 18, we get Equation 5.

$$\begin{aligned}
P(\mathcal{A}) = &\int_0^\infty \int_0^\infty \frac{e^{-\theta q_x - \lambda(T-t_x)}}{1 - e^{-\theta q_x}\left(1 - e^{-\lambda(T-t_x)}\right)} \frac{(a + t_x - t_1)^{r+x-1}}{\Gamma(r+x-1)} \lambda^{r+x-2} e^{-\lambda(a+t_x-t_1)} \\
&\times \frac{(b + B_{x-1})^s}{\Gamma(s)} \theta^{s-1} e^{-\theta(b+B_{x-1})} \frac{1 - e^{-\theta q_x}\left(1 - e^{-\lambda(T-t_x)}\right)}{1 - \left(\frac{b+B_{x-1}}{b+B_x}\right)^s \left[1 - \left(\frac{a+t_x-t_1}{a+T-t_1}\right)^{r+x-1}\right]} d\lambda\, d\theta \\
= &\frac{(a+t_x-t_1)^{r+x-1}}{\Gamma(r+x-1)} \frac{(b+B_{x-1})^s}{\Gamma(s)} \left[1 - \left(\frac{b+B_{x-1}}{b+B_x}\right)^s \left[1 - \left(\frac{a+t_x-t_1}{a+T-t_1}\right)^{r+x-1}\right]\right]^{-1} \\
&\times \int_0^\infty \lambda^{r+x-2} e^{-\lambda(a+T-t_1)} d\lambda \int_0^\infty \theta^{s-1} e^{-\theta(b+B_x)} d\theta \\
= &\left(\frac{a+t_x-t_1}{a+T-t_1}\right)^{r+x-1} \left(\frac{b+B_{x-1}}{b+B_x}\right)^s \left[1 - \left(\frac{b+B_{x-1}}{b+B_x}\right)^s \left[1 - \left(\frac{a+t_x-t_1}{a+T-t_1}\right)^{r+x-1}\right]\right]^{-1} \\
= &\left[1 - \left(\frac{a+T-t_1}{a+t_x-t_1}\right)^{r+x-1} \left(1 - \left(\frac{b+B_x}{b+B_{x-1}}\right)^s\right)\right]^{-1}
\end{aligned} \tag{20}$$

Through some straight-forward, but tedious, manipulation of terms, we can also write $P(\mathcal{A})$ in terms of the marginal likelihood.

$$P(\mathcal{A}) = \frac{1}{\mathcal{L}} \frac{\Gamma(r+x-1)}{\Gamma(r)} \frac{a^r}{(a+T-t_1)^{r+x-1}} \left(\frac{b}{b+B_x}\right)^s \tag{21}$$

Now we can compute the expected number of transactions for a specific customer, given an observed transaction history. Let $X(t^*)$ be the number of purchases in the *next* period of duration t^* (i.e., during the interval from T to $T + t^*$). Given a customer's observed history and individual-level parameters, the expected number of orders during the next t^* weeks is the probability of still being alive at time T, times the prior expectation in Equation 17.

$$E[X(t^*)|x,,t_x,\theta,\lambda] = \frac{e^{-\theta e^{\beta' z_x} - \lambda(T-t_x)}}{1 - e^{-\theta e^{\beta' z_x}}(1 - e^{-\lambda(T-t_x)})} \sum_{k=1}^{\infty} \frac{\gamma(k,\lambda t^*)}{\Gamma(k)} e^{-\theta B_k} \tag{22}$$

Equation 6 comes from integrating λ and θ in Equation 22 over the posterior density in Equation 18.

$$\begin{aligned}
E[X(t^*)|x,t_x] &= \frac{(a+t_x-t_1)^{r+x-1}}{\Gamma(r+x-1)} \frac{(b+B_{x-1})^s}{\Gamma(s)} \left[1 - \left(\frac{b+B_{x-1}}{b+B_x}\right)^s \left[1 - \left(\frac{a+t_x-t_1}{a+T-t_1}\right)^{r+x-1}\right]\right]^{-1} \\
&\quad \times \sum_{k=1}^{\infty} \int_0^{\infty} \frac{\gamma(k,\lambda t^*)}{\Gamma(k)} \lambda^{r+x-2} e^{-\lambda(a+T-t_1)} d\lambda \int_0^{\infty} \theta^{s-1} e^{-\theta(b+B_x-B_k)}\, d\theta \\
&= \left(1 - \left(\frac{b+B_{x-1}}{b+B_x}\right)^s \left[1 - \left(\frac{a+t_x-t_1}{a+T-t_1}\right)^{r+x-1}\right]\right)^{-1} \\
&\quad \times \sum_{k=1}^{\infty} \frac{(a+t_x-t_1)^{r+x-1}}{\Gamma(r+x-1)\Gamma(k)} \left(\frac{B_x+b}{B_x+B_k+b}\right)^s \frac{t^{*k}\Gamma(r+x+k-1)}{k(t^*+a+T-t_1)^{r+x+k-1}} \\
&\quad \times {}_2F_1\left(1, r+x+k-1; k+1; \frac{t^*}{t^*+a+T-t_1}\right) \\
&= P(\mathcal{A}) \times \sum_{k=1}^{\infty} \left(\frac{B_x+b}{B_x+B_k+b}\right)^s \widetilde{\mathbb{B}}\left(\frac{t^*}{t^*+a+T-t_1}; k, r+x-1\right)
\end{aligned} \tag{23}$$

Appendices B and C are available in the online supplement.

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